Quiz on Friday

Section 23 + Elasticity of Demand eqn

Section 24: Elasticity of demand

FACT: if price goes ↑, then demand ↓

Elasticity of demand = $\dfrac{\text{percent change in demand}}{\text{percent change in price}}$

→ a measure of how the demand (of a product) changes in response to a change in price

Why "percent" change?

Stock A: bought for \$10 → inc by 50 ¢

Stock B: bought for \$2 → inc by 20 ¢

better!

Stock A: $\frac{.50}{10} = 5\%$

Stock B: $\frac{.20}{2} = 10\%$

We're going to focus on relative change!

What if p is changed by a **small amount**?

→ this means going from p to $p+h$

Suppose we have a product at price p with demand $q = q(p)$.

- What is the percent change in p?

→ percent change → $\frac{h}{p} \times 100$

- What is the percent change in demand?

→ percent change $= \frac{q(p+h) - q(p)}{q(p)} \times 100$

Back to elasticity:

$$\text{Elasticity} = \frac{\text{Percent change in demand}}{\text{percent change in price}}$$

$$= \left[\frac{q(p+h) - q(p)}{q(p)} \cancel{\times 100}\right] \left(\frac{p}{h \cancel{\times 100}}\right)$$

$$= \frac{-\left[q(p) - q(p+h)\right]}{h} \cdot \frac{p}{q(p)}$$

As $h \to 0$,

$$E(p) = -\left[\frac{dq}{dp}\right] \cdot \frac{p}{q}$$

$$E(p) = \frac{-p}{q(p)} \left[\frac{dq}{dp}\right]$$

$\Rightarrow$ Recall: Revenue = (price) $\times$ (demand)

$$R(p) = p \cdot q(p)$$

Let's differentiate:

$$R'(p) = 1 q(p) + p \cdot q'(p)$$

$$= q(p) \left[1 + \frac{p}{q(p)} \cdot q'(p) \right]$$

$$R'(p) = q(p) \left[1 - E(p) \right]$$

$\Rightarrow$ If $E(p) < 1$, then $R'(p) > 0$

i.e. if $E < 1$, then rev increases

inelastic

$\Rightarrow$ If $E(p) > 1$, then $R'(p) < 0$

i.e. If $E > 1$, then rev. decreases

elastic

$\Rightarrow$ If $E(p) = 1$, then $R'(p) = 0$

i.e If $E = 1$, then rev is max/min.

unit elasticity

Let's look at "ratio version" of $E(p)$:

$$E(p) = \frac{\%\,\Delta \text{ in demand}}{\%\,\Delta \text{ in price}} \quad < \text{ or } > \text{ or } = \quad 1$$

$$\%\,\Delta \text{ in demand} \quad < \text{ or } > \text{ or } = \quad \%\,\Delta \text{ in price}$$

Example 24.1. For a certain product, it is known that the relationship between price and demand is given by $p = -.02q + 400$, where $0 \leq q \leq 20{,}000$. (a) Find the elasticity function, $E(p)$. (b) Compute $E(100)$ and interpret the result. (c) Compute $E(300)$ and interpret the result. (d) At what price do we have unit elasticity of demand?

a)

$$\boxed{E(p) = \frac{-p}{q(p)} \cdot q'(p)}$$

$p = -.02q + 400$

$\Longrightarrow$ Need a function for q!

$$\Longrightarrow \quad q(p) = -50p + 20000$$

Finding $q'(p)$: $\quad q'(p) = -50$

So,

$$E(p) = \left(\frac{-p}{-50p + 20000}\right)(-50)$$

$$= \frac{50p}{-50p + 20000} = \frac{p}{400-p}$$

b) $E(p) = \frac{p}{400-p}$

$$E(100) = \frac{100}{400-100} = \frac{100}{300} = 1/3$$

Since $E < 1 \implies$ Revenue would have a slight increase if price increases

c) $E(300) = \frac{300}{400-300} = \frac{300}{100} = 3 > 1$

Since $E > 1 \implies$ Revenue would decrease slightly if price increases slightly.

d) $E(p) = 1$

$$\frac{p}{400-p} = 1 \implies p = 400-p$$

$$2p = 400$$

$$p = 200$$

If $p = \$200$, then rev is maximized!

1. Suppose that the demand equation for a certain commodity is $q = 60 - p$ (for $0 \leq p \leq 60$).

   (a) Express the elasticity of demand as a function of p.

   (b) Calculate the elasticity of demand when the price is $p = 20$. Interpret your answer.

   (c) At what price is the elasticity of demand equal to 1?

a)

$$E(p) = \frac{-p}{q(p)} \cdot q'(p)$$

$$q(p) = 60 - p$$

$$\Rightarrow q'(p) = -1$$

$$E(p) = \frac{-p}{60-p}(-1) = \frac{p}{60-p}$$

b) $$E(20) = \frac{20}{60-20} = \frac{20}{40} = \frac{1}{2} < 1$$

$\Rightarrow$ Since $E(20) < 1$, a slight increase in price (from \$20) results in a slight increase in revenue.

c) $$E(p) = 1 \Rightarrow \frac{p}{60-p} = 1$$

$$\Rightarrow p = 60 - p$$

$$\Rightarrow 2p = 60$$

$$\Rightarrow p = 30$$

$p = \$30$ is the price that gives max revenue.

8. The owner of the Showplace video store has estimated that the rental price p (in dollars) of new-release DVDs is related to the quantity q (in thousands) rented each day by the demand equation $p = \frac{3}{2}\sqrt{16 - q^2}$.

(a) What is the elasticity function $E(p)$ for this demand function?

(b) If the store owner increased the current \$4 price slightly, can she expect her revenue to increase or decrease?

(c) Use your elasticity function to determine the price that will yield the maximum revenue. What is the maximum revenue?

$$E(p) = \frac{-p}{q(p)} \cdot q'(p)$$

$$p = \frac{3}{2}\sqrt{16 - q^2}$$

$$\Rightarrow \frac{2p}{3} = \sqrt{16 - q^2}$$

$$\Rightarrow \frac{4p^2}{9} = 16 - q^2$$

$$\Rightarrow q^2 = 16 - \frac{4p^2}{9}$$

$$\Rightarrow q = \sqrt{16 - \frac{4q^2}{9}}$$

Now,

$$q'(p) = \frac{1}{2}\left(16 - \frac{4p^2}{9}\right)^{-1/2} \cdot \left(-\frac{8p}{9}\right)$$

$$E(p) = \frac{-p}{q(p)} \cdot q'(p)$$

$$= \frac{-p}{\sqrt{16 - \frac{4p^2}{9}}} \cdot \frac{1}{2}\left(16 - \frac{4p^2}{9}\right)^{-1/2} \cdot \left(\frac{-8p}{9}\right)$$

$$= \frac{4}{9} p^2 \frac{1}{\sqrt{16 - \frac{4p^2}{9}}} \cdot \frac{1}{\sqrt{16 - \frac{4p^2}{9}}}$$

$$= \frac{4p^2}{9\left(16 - \frac{4p^2}{9}\right)}$$

$$= \frac{p^2}{9\left(4 - \frac{p^2}{9}\right)} = \frac{p^2}{36 - p^2}$$

b) $E(4) = \frac{16}{36-16} = \frac{16}{20} < 1$

A slight increase in price (from 4) results in an inc. in rev.

c) We want $E(p) = 1$:

$$\frac{p^2}{36-p^2} = 1 \implies p^2 = 36 - p^2$$

$$\implies p^2 = 18$$

$$\implies p = 3\sqrt{2} \approx \$4.24$$