

# Quiz on Friday

## Section 23 + Elasticity of Demand eqn

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### Section 24: Elasticity of demand

**FACT:**

if price goes  $\uparrow$ ,  
then demand  $\downarrow$

$$\text{Elasticity of demand} = \frac{\text{Percent change in demand}}{\text{percent change in price}}$$

→ a measure of how the demand (of a product) changes in response to a change in price

Why "percent" change?

Stock A: bought for \$10 → inc by 50¢

Stock B: bought for \$2 → inc by 20¢



Stock A:  $\frac{.50}{10} = 5\%$

Stock B:  $\frac{.20}{2} = 10\%$

better!

We're going to focus on relative change!

What if  $p$  is changed by a small amount?

→ this means going from  $p$  to  $p+h$

Suppose we have a product at price  $p$  with demand  $q = q(p)$ .

• What is the percent change in  $p$ ?

→ percent change →  $\frac{h}{p} \times 100$

• What is the percent change in demand?

→ percent change =  $\frac{q(p+h) - q(p)}{q(p)} \times 100$

Back to elasticity:

$$\text{Elasticity} = \frac{\text{Percent change in demand}}{\text{percent change in price}}$$

$$= \left[ \frac{q(p+h) - q(p)}{q(p)} \times 100 \right] \left( \frac{p}{h} \times 100 \right)$$

$$= \frac{-[q(p) - q(p+h)]}{h} \cdot \frac{p}{q(p)}$$

As  $h \rightarrow 0$ ,

$$E(p) = - \left[ \frac{dq}{dp} \right] \cdot \frac{p}{q}$$

$$E(p) = \frac{-p}{q(p)} \left[ \frac{dq}{dp} \right]$$

⇒ Recall: Revenue = (price) × (demand)

$$R(p) = p \cdot q(p)$$

let's differentiate:

$$R'(p) = 1 \cdot q(p) + p \cdot q'(p)$$

$$= q(p) \left[ 1 + \frac{p \cdot q'(p)}{q(p)} \right]$$

$$R'(p) = q(p) [1 - E(p)]$$

⇒ If  $E(p) < 1$ , then  $R'(p) > 0$

i.e. if  $E < 1$ , then rev increases

**inelastic**

⇒ If  $E(p) > 1$ , then  $R'(p) < 0$

i.e. if  $E > 1$ , then rev. decreases

**elastic**

⇒ If  $E(p) = 1$ , then  $R'(p) = 0$

i.e. if  $E = 1$ , then rev is max/min.

**unit elasticity**

Let's look at "ratio version" of  $E(p)$ :

$$E(p) = \frac{\% \Delta \text{ in demand}}{\% \Delta \text{ in price}}$$

<  
or  
>  
or  
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1

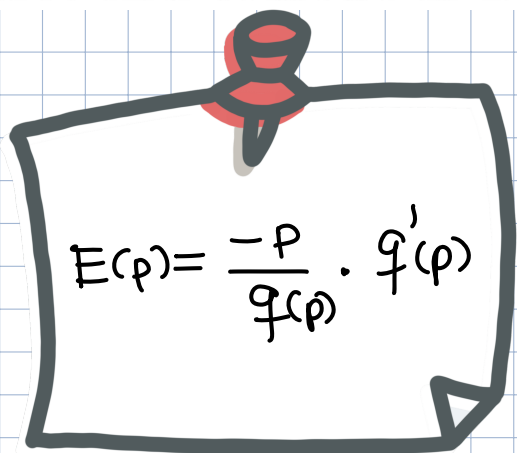
%  $\Delta$  in demand

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%  $\Delta$  in price

**Example 24.1.** For a certain product, it is known that the relationship between price and demand is given by  $p = -.02q + 400$ , where  $0 \leq q \leq 20,000$ . (a) Find the elasticity function,  $E(p)$ . (b) Compute  $E(100)$  and interpret the result. (c) Compute  $E(300)$  and interpret the result. (d) At what price do we have unit elasticity of demand?

a)


$$E(p) = \frac{-p}{q(p)} \cdot q'(p)$$

$$p = -.02q + 400$$

$\Rightarrow$  Need a function for  $q$ !

$$\Rightarrow q(p) = -50p + 20000$$

Finding  $q'(p)$ :

$$q'(p) = -50$$

So,

$$E(p) = \left( \frac{-p}{-50p + 20000} \right) (-50)$$

$$= \frac{50p}{-50p + 20000} = \frac{p}{400 - p}$$

$$b) E(p) = \frac{p}{400 - p}$$

$$E(100) = \frac{100}{400 - 100} = \frac{100}{300} = \frac{1}{3}$$

Since  $E < 1 \Rightarrow$  Revenue would have a slight increase if price increases

$$c) E(300) = \frac{300}{400 - 300} = \frac{300}{100} = 3 > 1$$

Since  $E > 1 \Rightarrow$  Revenue would decrease slightly if price increases slightly.

$$d) E(p) = 1$$

$$\frac{p}{400 - p} = 1 \Rightarrow p = 400 - p$$

$$2p = 400$$

$$p = 200$$

If  $p = \$200$ , then rev is maximized!

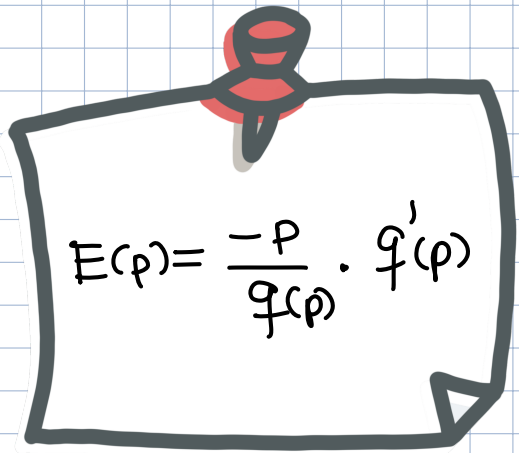
1. Suppose that the demand equation for a certain commodity is  $q = 60 - p$  (for  $0 \leq p \leq 60$ ).

(a) Express the elasticity of demand as a function of  $p$ .

(b) Calculate the elasticity of demand when the price is  $p = 20$ . Interpret your answer.

(c) At what price is the elasticity of demand equal to 1?

a)


$$E(p) = \frac{-p}{q(p)} \cdot q'(p)$$

$$q(p) = 60 - p$$

$$\Rightarrow q'(p) = -1$$

$$E(p) = \frac{-p}{60-p} (-1) = \frac{p}{60-p}$$

$$b) E(20) = \frac{20}{60-20} = \frac{20}{40} = \frac{1}{2} < 1$$

$\Rightarrow$  Since  $E(20) < 1$ , a slight increase in price (from \$20) results in a slight increase in revenue.

$$c) E(p) = 1 \Rightarrow \frac{p}{60-p} = 1$$

$$\Rightarrow p = 60 - p$$

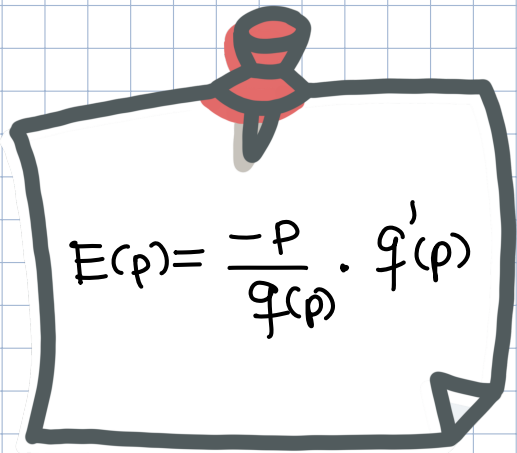
$$\Rightarrow 2p = 60$$

$$\Rightarrow p = 30$$

$p = \$30$  is the price that gives max revenue.

8. The owner of the Showplace video store has estimated that the rental price  $p$  (in dollars) of new-release DVDs is related to the quantity  $q$  (in thousands) rented each day by the demand equation  $p = \frac{3}{2}\sqrt{16 - q^2}$ .

- What is the elasticity function  $E(p)$  for this demand function?
- If the store owner increased the current \$4 price slightly, can she expect her revenue to increase or decrease?
- Use your elasticity function to determine the price that will yield the maximum revenue. What is the maximum revenue?


$$E(p) = \frac{-p}{q(p)} \cdot q'(p)$$

$$p = \frac{3}{2}\sqrt{16 - q^2}$$

$$\Rightarrow \frac{2p}{3} = \sqrt{16 - q^2}$$

$$\Rightarrow \frac{4p^2}{9} = 16 - q^2$$

$$\Rightarrow q^2 = 16 - \frac{4p^2}{9}$$

$$\Rightarrow q = \sqrt{16 - \frac{4p^2}{9}}$$

Now,

$$q'(p) = \frac{1}{2} \left( 16 - \frac{4p^2}{9} \right)^{-1/2} \cdot \left( -\frac{8p}{9} \right)$$

$$E(p) = \frac{-p}{q(p)} \cdot q'(p)$$

$$= \frac{-p}{\sqrt{16 - \frac{4p^2}{9}}} \cdot \frac{1}{2} \left( 16 - \frac{4p^2}{9} \right)^{-1/2} \cdot \left( -\frac{8p}{9} \right)$$



$$= \frac{4}{9} p^2 \frac{1}{\sqrt{16 - \frac{4p^2}{9}}} \cdot \frac{1}{\sqrt{16 - \frac{4p^2}{9}}}$$

$$= \frac{4p^2}{9 \left(16 - \frac{4p^2}{9}\right)}$$

$$= \frac{p^2}{9 \left(4 - \frac{p^2}{9}\right)} = \frac{p^2}{36 - p^2}$$

$$b) \quad E(4) = \frac{16}{36 - 16} = \frac{16}{20} < 1$$

A slight increase in price (from 4) results in an inc. in rev.

c) We want  $E(p) = 1$  :

$$\frac{p^2}{36 - p^2} = 1 \quad \Rightarrow \quad p^2 = 36 - p^2$$

$$\Rightarrow \quad p^2 = 18$$

$$\Rightarrow \quad p = 3\sqrt{2} \approx \$4.24$$