Quiz Friday (Section 23)

Section 23: Optimization

Types of Opt. Problems:

I. **Basic** (Single function)

$\Rightarrow$ the "function to be optimized" is ready to go.

Ex: Cost: $C(x) = x^2 + 3x + 12$

Revenue: $R(x) = 3x^2 + x - 2$

Maximize Profit.

II. **Geometric** (Volume / SA / area)

$\rightarrow$ You'll usually have 2 functions

1. function to be optimize
2. Constraint

$\Rightarrow$ This type almost always requires use of constraint to eliminate a variable*

* Substitution only happens if fct to be optimized depends on mult. variables

III. Linear Programming

$\Rightarrow$ Ex: Rebate problem (down below)

(linear factor) $\times$ (linear factor)

Example 23.5. A manufacturer has been selling television sets. He sells 1,000 TVs per week if the price is \$450 each. A survey tells him that for each \$1 rebate he offers, the number of sets sold (q) will increase by 10 per week. (a) How much rebate should he offer in order to maximize revenue? (b) How much rebate should he offer in order to maximize profit if the cost function is $C(q) = 68,000 + 150q$?

(a) Want: maximize Revenue

Known: Revenue = (price) $\times$ (demand)

Let X = rebate amount

$$\left.\begin{aligned} \text{Price} &= 450 - x \\ \text{Demand} &= 1000 + 10x \end{aligned}\right\} R(x) = (450-x)(1000+10x)$$

$$\Rightarrow R(x) = 450,000 - 1000x + 4500x - 10x^2$$

$$= 450,000 + 3500x - 10x^2$$

$\longrightarrow$ a fct. of one-variable

Crit. Pts: $R'(x) = 3500 - 20x$

$$\Rightarrow x = 175$$

Verify $x = 175$ is a local max:

SDT: $R''(x) = -20 \Rightarrow$ always concave down

By SDT, $x = 175$ is a local max.

Price $= \$275$

(b) $C(q) = 68,000 + 150q$?

Want: maximize profit

Known: Cost: $C(q) = 68000 + 150q$

$q =$ demand $= 1000 + 10x$

Profit = Revenue − Cost

$$P(x, q) = [450,000 + 3500x - 10x^2] - [68000 + 150q]$$

$\to$ P depends on x and q!

Make sure that Profit only depends on one-variable!

Sub in for $q = 1000 + 10x$

$$P(x) = [450{,}000 + 3500x - 10x^2] - (68000 + 150[1000 + 10x])$$

$$P(x) = 232000 + 2000x - 10x^2$$

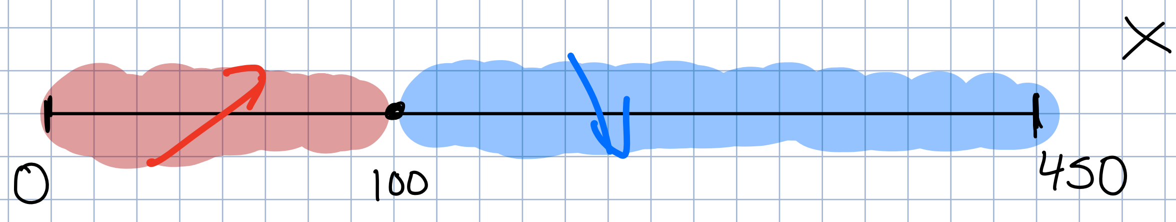
Crit. Pts: $P'(x) = 2000 - 20x$

$$\Rightarrow x = 100$$

Verification:

$x \in [0, 450]$

FDT



Alternatively,

$P(0) = ?$ $P(450) = ?$

$P(100) = ?$

1. Find non-negative numbers x and y such that $x + y = 150$ and x^2y is maximized.

Want: maximize $f(x,y) = x^2y$

Known: $x+y=150$

$x,y \geq 0$

$\Rightarrow$ use "$x+y=150$" to eliminate a variable:

$x+y=150$

$\Rightarrow \quad y = 150 - x$

Since $f(x,y) = x^2y$

$\Rightarrow \quad f(x) = x^2(150-x)$

$= 150x^2 - x^3$

Crit. Pts: $f'(x) = 300x - 3x^2$

$300x - 3x^2 = 0$

$3x(100-x) = 0$

$x=0$	$x=100$

Verify: SDT $f''(x) = 300 - 6x$

$f''(0) = 300$

$f''(100) = 300 - 600 = -300$

By SDT $x = 100$ is a local max

4. If a manufacturer charges $p(x)$ dollars per item, where $p(x) = 4 - \frac{x}{12}$, then x thousand items will be sold.

(a) Find an expression for the total revenue from the sale of x thousand items.

(b) Find the value of x that leads to maximum revenue.

(c) Find the maximum revenue.

a) Revenue = (price)×(demand)

$= \left(4 - \frac{x}{12}\right) x$

$\Rightarrow R(x) = 4x - \frac{x^2}{12}$

b) **Want:** maximize $R(x)$

Known: $R(x) = 4x - \frac{x^2}{12}$

Crit. Pts: $R'(x) = 4 - \frac{x}{6}$

$R'(x) = 0 \Rightarrow x = 24$

Verify: $R''(x) = -1/6 \Rightarrow$ always concave down

By SDT, $x=24$ is a local max

c) $R(24) = 4(24) - \frac{(24)^2}{12}$

$= 96 - 48 = \$48$