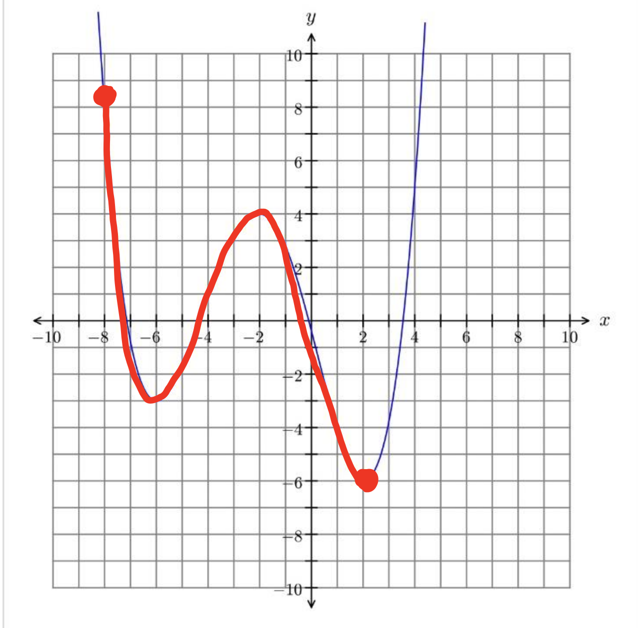
Review problems for Exam 2 Math 220

1. Give the *ordered pairs* of the extremes on intervals named. If feature is absent, write none:



On $(-\infty,\infty)$:

Local maxima: (−2, 4)

Local minima: (−6,−3), (2, −6)

Absolute maxima: N/A

Absolute minima: (2, −6)

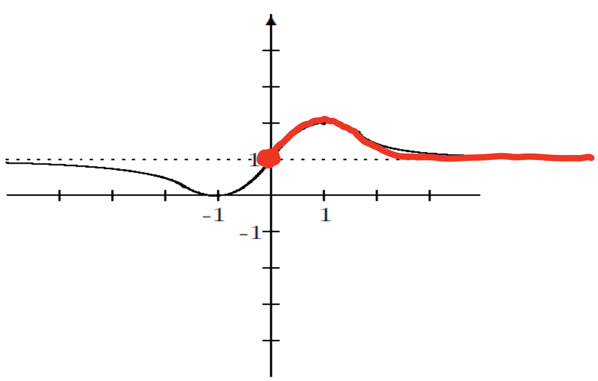
On $[-8,2]$:

Local maxima: (−2, 4), (−8, 8)

Local minima: (−6, −3), (2, −6)

Absolute maxima: (−8, 8)

AbSolute minima: (2, −6)



On $(-\infty,\infty)$:

Local maxima: (1, 2)

Local minima: (−1, 0)

Absolute maxima: (1, 2)

Absolute minima: (−1, 0)

On $[0,\infty)$:

Local maxima: (1, 2)

Local minima: (0, 1)

Absolute maxima: (1, 2)

Absolute minima: (0, 1)

2. Find the equations of the lines tangent to the curve: $2e^x = y^2 - x$, at $x = 0$.
3. Given the curve $x^2 + 3y^2 = 22$:
   a) What two values of y does the curve attain when $x = 2$?
   b) Find the equation of the line tangent to the positive value of y that you found in (a). You must use implicit differentiation to find the slope.

4. The demand equation for a product shows us that the quantity produced varies with the price according to the equation $q = 1200/p$. The price is increasing at a rate of \$0.06 per month. How fast is the demand for this product changing when the price is \$6.00? Simplify to reduced form.
5. $R(x) = 50x - \frac{1}{2}x^2$; $C(x) = 4x + 10$. Revenue and cost are in dollars.

   a) Find the *rate* at which profit is changing when $x = 10$ and $dx/dt = 5$ units per day.
   b) Draw a graph of the profit function. At what value of x (level of sales) is profit at a maximum?
6. A pole 13 ft long leans against a vertical wall. If the lower end is moving away from the wall at the rate of 0.4 ft/sec, how fast is the upper end coming down when the lower end is 12 ft from the wall?
7. a) A pebble is dropped into a pool of water, generating circular ripples. The radius of the largest ripple is increasing at a constant rate of 6 inches per second. What is the increase in the area of the big ripple circle after 3 seconds have passed? (HINT: You will need to find the radius length at time 3 seconds).
   b) What is the increase in the circumference of the ripple after 3 seconds have passed?
8. Given the function $f(x) = 2x^3 - x^4$, answer each of the questions, showing all your work. *Answer with interval notation for domain and other features.*
   Write 'none' or 'nowhere' where appropriate.
   a) D_f:

   b) Intercepts:

   c) End behavior; that is, $\lim_{x\to-\infty} f(x) =$ ____ and $\lim_{x\to\infty} f(x) =$ _____

   d) $f'(x) =$ $\qquad\qquad\qquad$ $f''(x) =$

   e) Critical numbers (x values only):

2. Find the equations of the lines tangent to the curve: $2e^x = y^2 - x$, at $x = 0$.

$$2e^x = y^2 - x \qquad \text{at } x=0$$

$\frac{dy}{dx}$ = slope! $\quad 2e^x = 2y\left(\frac{dy}{dx}\right) - 1$

$$\Rightarrow \frac{dy}{dx} = \frac{2e^x+1}{2y}$$

What value do we use for y?

$$2e^x = y^2 - x \quad , \quad x=0$$

$$\Rightarrow \quad 2e^0 = y^2 - 0$$

$$\Rightarrow \quad 2 = y^2 \quad \Rightarrow \quad y = \pm\sqrt{2}$$

First pt: $(0, \sqrt{2})$

$$\frac{dy}{dx} = \frac{2e^0+1}{2(\sqrt{2})} = \frac{2+1}{2\sqrt{2}} = \frac{3}{2\sqrt{2}}$$

$$y - y_0 = m(x - x_0)$$

$$y-\sqrt{2} = \frac{3}{2\sqrt{2}}(x-0)$$

Second pt: $(0,-\sqrt{2})$

$$\frac{dy}{dx} = \frac{2e^0+1}{2(-\sqrt{2})} = \frac{-3}{2\sqrt{2}}$$

$$y+\sqrt{2} = \frac{-3}{2\sqrt{2}}(x-0)$$

3. Given the curve $x^2 + 3y^2 = 22$:
   a) What two values of y does the curve attain when $x = 2$?
   b) Find the equation of the line tangent to the positive value of y that you found in (a). You must use implicit differentiation to find the slope.

$$x^2+3y^2 = 22$$

Find y!

$$4+3y^2 = 22$$

$$3y^2 = 18$$

$$y^2 = 6$$

$$\Rightarrow y = \pm\sqrt{6}$$

$(2, \sqrt{6}) \quad \Rightarrow \quad 2x + 6y\left(\frac{dy}{dx}\right) = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{3y}$$

So,

$$m = \left.\frac{dy}{dx}\right|_{\substack{x=2 \\ y=\sqrt{6}}} = \frac{-2}{3\sqrt{6}}$$

$$y - y_0 = m(x - x_0)$$

$$y - \sqrt{6} = \frac{-2}{3\sqrt{6}}(x - 2)$$

4. The demand equation for a product shows us that the quantity produced varies with the price according to the equation $q = 1200/p$. The price is increasing at a rate of \$0.06 per month. How fast is the demand for this product changing when the price is \$6.00? Simplify to reduced form.

q = quantity prod.

p = price

Known:

$$\frac{dp}{dt} = .06$$

$$p = 6$$

Goal:

$$\frac{dq}{dt} = ???$$

Rel. Eqn:

$$q = \frac{1200}{p} = 1200p^{-1}$$

$$\Rightarrow \quad \frac{dq}{dt} = -1200\,p^{-2}\left(\frac{dp}{dt}\right)$$

$$\frac{dq}{dt} = \frac{-1200}{(6)^2}\,(.06)$$

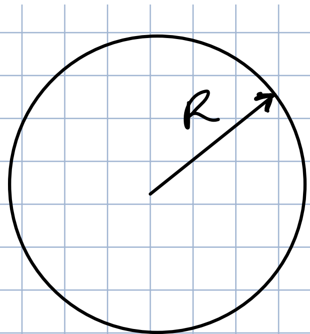
$$= \frac{-1200}{36}\left(\frac{6}{100}\right)$$

$$= \frac{-12}{36} \cdot 6$$

$$= \frac{-12}{6} = -2 \text{ units/month}$$

7. a) A pebble is dropped into a pool of water, generating circular ripples. The radius of the largest ripple is increasing at a constant rate of 6 inches per second. What is the increase in the area of the big ripple circle after 3 seconds have passed? (HINT: You will need to find the radius length at time 3 seconds).

b) What is the increase in the circumference of the ripple after 3 seconds have passed?



R = radius

A = area

Known:

$$\frac{dR}{dt} = 6$$

$R\left(\begin{matrix}\text{3 secs} \\ \text{after start}\end{matrix}\right)$: 18

Goal:

dA

$\frac{}{dt}$

Rel. eqn:

$A = \pi R^2$

$$\frac{dA}{dt} = \pi 2R\left(\frac{dR}{dt}\right)$$

$$= \pi\, 2(18)(6)$$

$$= \pi\,(12)(18)$$

$$= \pi\,(180) + \pi(36)$$

$$= 216\pi \text{ in}^2/\text{sec}$$

b) **Rel eqn**

$C = 2\pi R$

$$\Rightarrow \frac{dC}{dt} = 2\pi\left(\frac{dR}{dt}\right)$$

$$= 2\pi(6) = 12\pi \quad \text{in}/\text{sec}$$

8. Given the function $f(x) = 2x^3 - x^4$, answer each of the questions, showing all your work.

Answer with interval notation for domain and other features.

Write 'none' or 'nowhere' where appropriate.

$f(x) = 2x^3 - x^4$
$= x^3(2-x)$

a) D_f: $(-\infty, \infty)$

b) Intercepts: $(0,0)$ $(2,0)$

c) End behavior; that is, $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -\infty$

d) $f'(x) = 6x^2 - 4x^3$ $\qquad f''(x) = 12x - 12x^2$

$12x(1-x)$

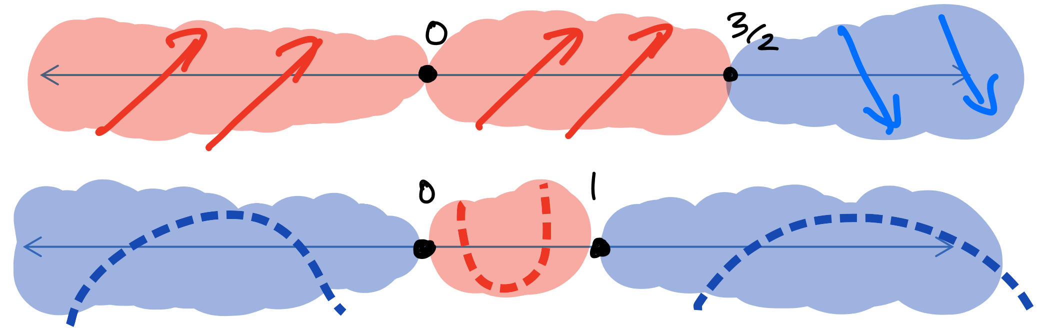
$x=0$
$x=1$

e) Critical numbers (x values only):

$2x^2(3-2x)$

$x=0$, $x=3/2$

f) Use the number lines for the sign analysis:



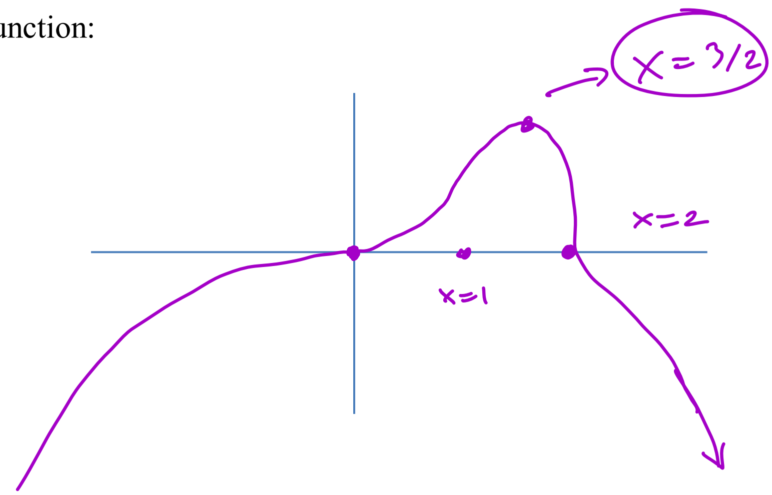
g) f increases on $(-\infty,0)\cup(0,3/2)$ f decreases on $(3/2,\infty)$

h) Local maximum $3/2$ Local minimum none (ordered pairs)

i) f is concave up on ________________ f is concave down on ________________

i) f has a point of inflection at $x=0$, $x=1$ (ordered pair)

j) Sketch the function:



9. Consider a function, and its derivatives: $f(x)=\dfrac{x^2-1}{3x^2}$, $f'(x)=\dfrac{2}{3x^3}$, $f''(x)=\dfrac{-2}{x^4}$

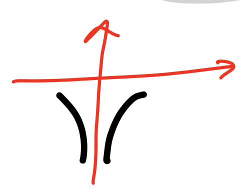
a) What are the intercepts of $f(x)$, in ordered pair form? $(1,0)$, $(-1,0)$

b) What is the vertical asymptote? $x=0$

c) Are there any critical numbers? NO Explain.

d) Are there any POI? NO Explain.

$x=0$ is not in domain



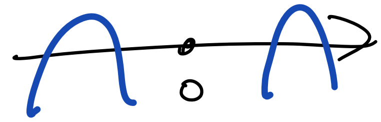
e) $\lim\limits_{x\to\infty}\dfrac{x^2-1}{3x^2}$ is an indeterminate form, upon inspection. *Without using a shortcut*, find this limit.

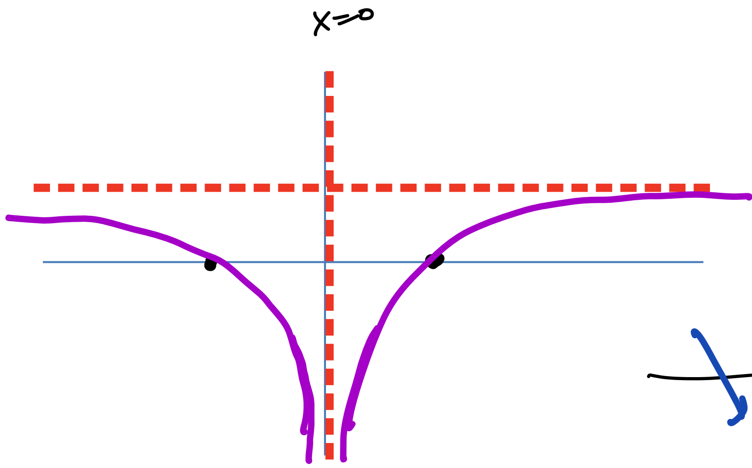
f) Hence, what is the horizontal asymptote?

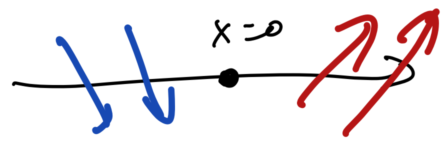
$$\lim_{x\to\infty}\frac{x^2-1}{3x^2}=\lim_{x\to\infty}\frac{1-\frac{1}{x^2}}{3}=1/3$$

$y=1/3$

g) Sketch a graph of this function with its features clearly marked:







10. a) The intermediate value theorem states that if a function f is continuous on a closed interval $[a, b]$ and if the sign of f changes (say positive to negative or vice versa) on $[a, b]$, then it has at least one real root on (a, b).

Verify that $f(x) = x^4 - 7x^3 + 4x - 1$ has at least one root between $x = -1$ and $x = 1$?

b) What theorem ensures that a continuous function on a closed interval is guaranteed to have an absolute maximum and an absolute minimum?

c) Draw a secant line from (0, 1) to (3, –2). Find the slope of this line? Then draw a tangent line to the graph that illustrates the mean value theorem. Thus, at the point of tangency $f'(x) =$ ___ ?