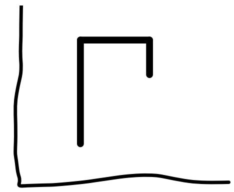
- Exam 2 10/27
- HW Due 10/25 on Sections 20/21.



Ex: Sketch a graph of

$$f(x) = \frac{-x^3+5x^2}{2x^2+4x+2} = \frac{-x^2(x-5)}{2(x+1)(x+1)}$$

Domain: $(-\infty,-1)\cup(-1,\infty)$

Roots: $(0,0)$, $(5,0)$

VA: (Check $x=-1$)

- $\lim\limits_{x\to -1^-} \dfrac{-x^2(x-5)}{2(x+1)(x+1)} \to \dfrac{6}{0^+} = +\infty$

- $\lim\limits_{x\to -1^+} \dfrac{-x^2(x-5)}{2(x+1)(x+1)} \to \dfrac{6}{0^+} = +\infty$

Conclusion: VA @ $x=-1$

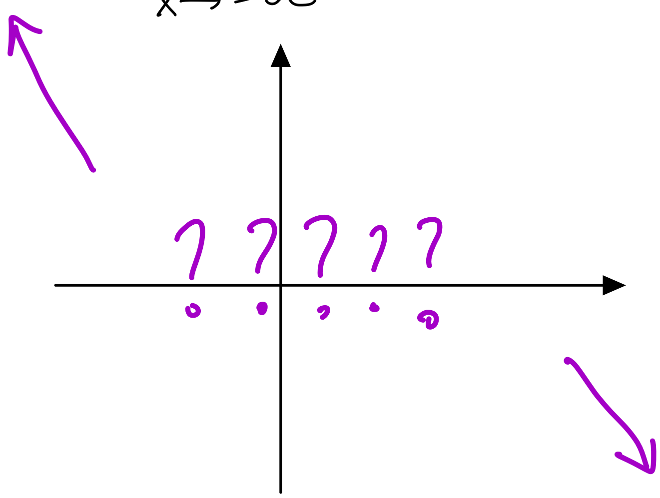
HA:

$$\lim_{x\to\infty} \frac{-x^3+5x^2}{2x^2+4x+2} = \lim_{x\to\infty} \frac{-x+5}{2+\frac{4}{x}+\frac{2}{x^2}}$$

$$= -\infty$$

$$\lim_{x\to-\infty} \frac{-x^2(x-5)}{2(x+1)(x+1)} = \lim_{x\to-\infty} \frac{-x\left(1-\frac{5}{x}\right)}{2\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)}$$

$$= \lim_{x\to-\infty} \frac{-x(1-0)}{2(1+0)(1+0)}$$



$$= \infty$$

Crit. Pts:

$$f'(x) = \frac{-x(x-2)(x+5)}{2(x+1)^3}$$

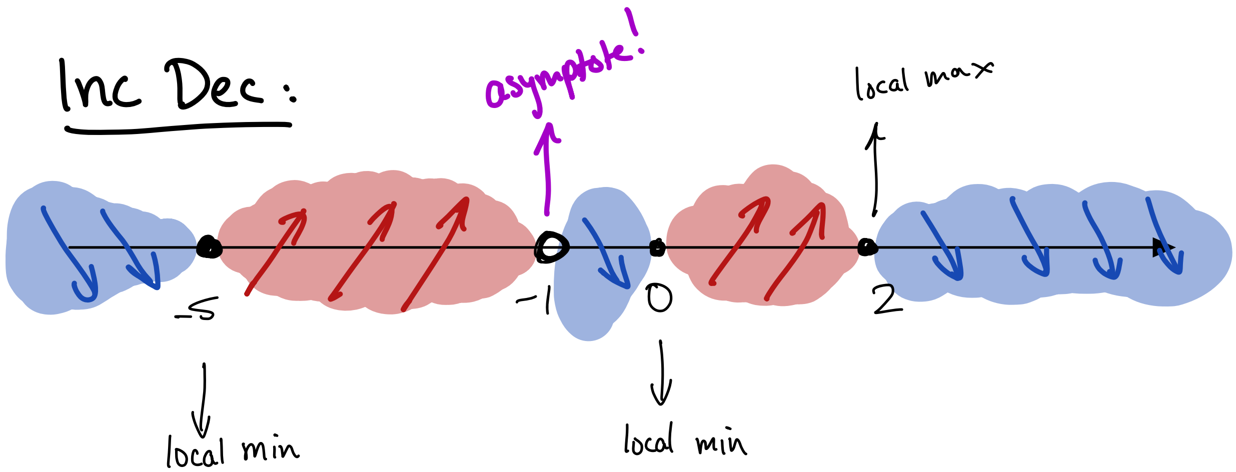
$x=0$

$x=2$

$x=-5$

$x=-1$

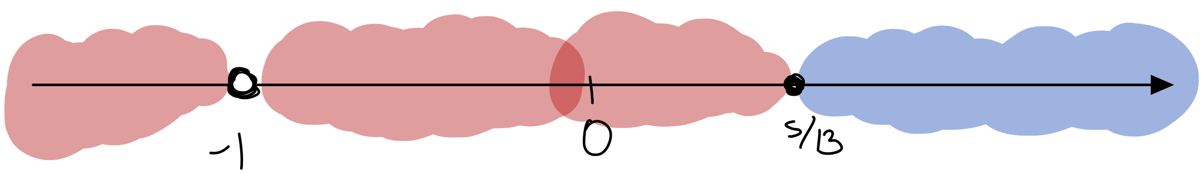
Inc Dec:

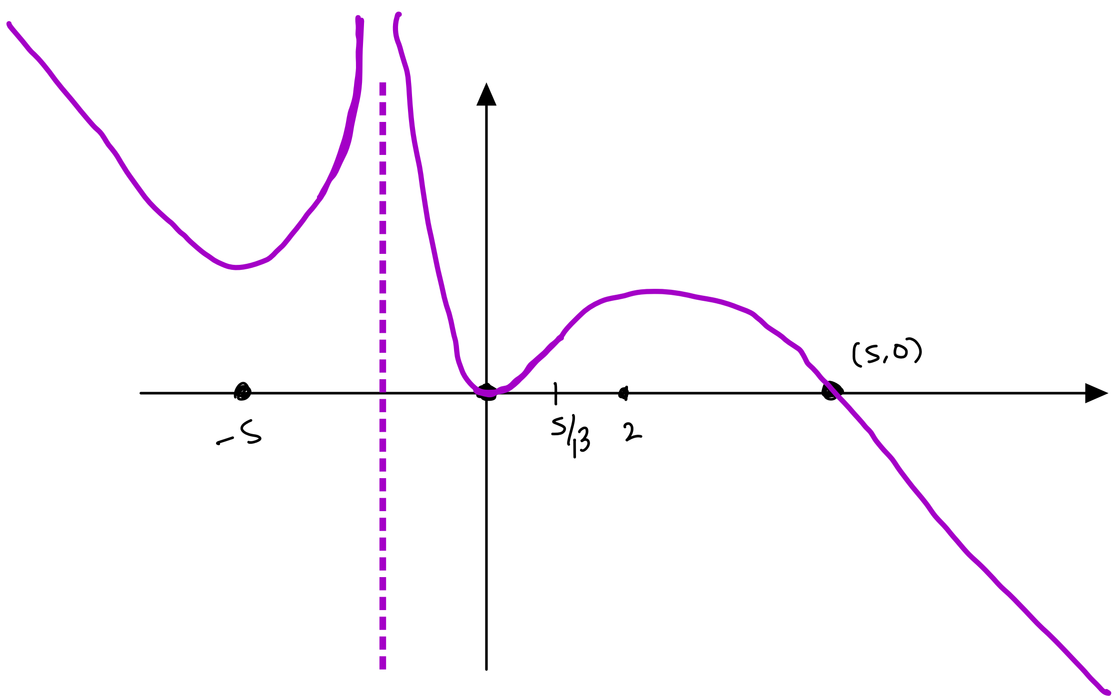


CU/CD:

$$f''(x) = \frac{5-13x}{(x+1)^4}$$

$x = 5/13$, $x = -1$





Section 22: Absolute extrema

Def^n (Absolute max/min)

Let f be a real-valued function. We say that f has an absolute max (or min) at $x=a$ if

- $f(x) \le f(a)$, for all x in domain (maximum)
- $f(x) \ge f(a)$, for all x in domain (minimum)

→ We don't always have abs. max/min.

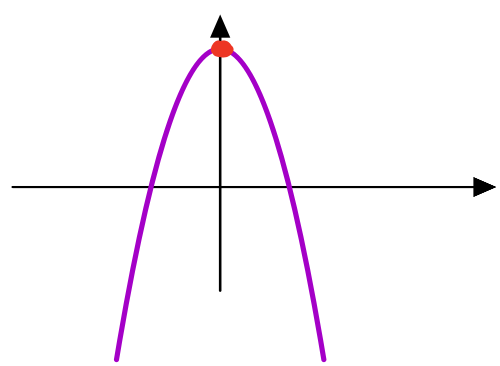
Let's look at $y = 4 - x^2$

→ We'll modify the domain and observe the corresp. effect on extrema.

a) Domain: $(-\infty, \infty)$

Abs. max: $x=0$, $f(0)=4$

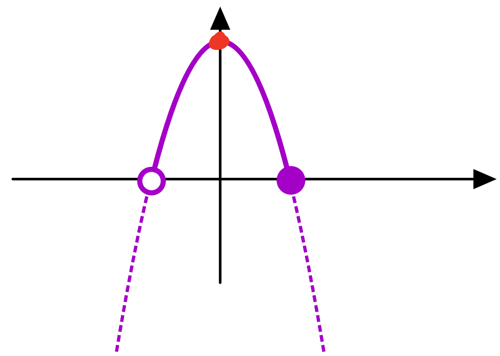
Abs. min: N/A



b) Domain: $(-2, 2]$

Abs. max: $x = 0$

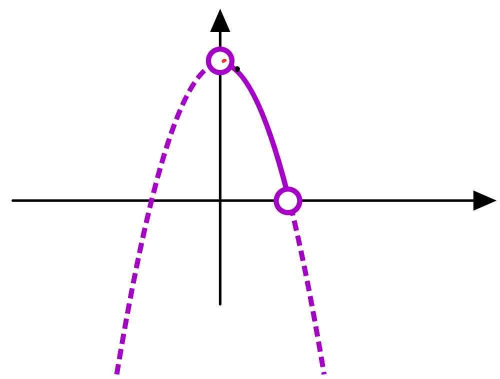
Abs. min: $x = 2$



c) Domain: $(0, 2)$

Abs max: N/A

Abs min: N/A



→ Note: the existence of abs. extrema depends on our given domain

⇒ Can we guarantee the existence of abs. extrema based on domain?

Theorem: (Extreme Value Theorem)

Let f be a real-valued continuous fct defined on $[a,b]$. Then f attains it abs max/min on $[a,b]$.

→ An abs. max and min must occur in $[a,b]$

→ From before, any extremum must be a crit. pt.

→ Using EVT, max/min may also occur at endpts.

Steps for finding abs. max/mins

1) Find all crit. pts in $[a,b]$

2) To classify the crit. pts, we evaluate them via $f(x)$.

3) Consider endpts as possible max/min.
   Pick largest as abs max
   Pick smallest as abs. min.

Ex: Find abs. extrema for

$$y = x^2 - 1 \quad \text{on} \quad [-1, 1]$$

→ $y' = 2x$

Crit pts: $x=0$

Pts to check: $x=0$, $x=1$, $x=-1$

$y(0)=-1$ $\longrightarrow$ smallest y-value

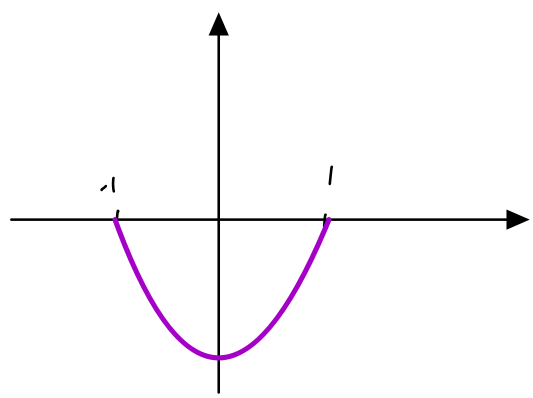
abs min @ $x=0$

$y(-1)=0$

$y(1)=0$

$\Big\}$ largest y-values

abs maxima @ $x=\pm 1$



Ex: Find abs max/min of

$$y=x^3 \quad \text{over} \quad [-1,2]$$

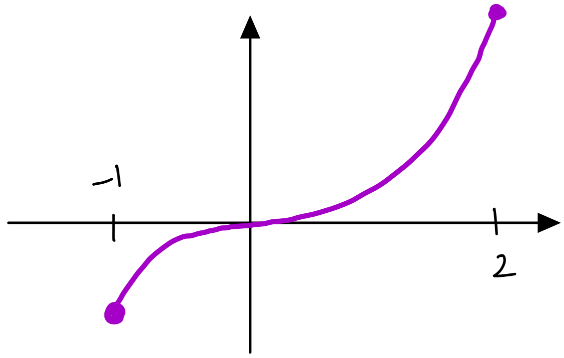
$\longrightarrow y'=3x^2$ Crit pts: $x=0$

Pts to check: $x=0$, $x=-1$, $x=2$

$y(0) = 0$

$y(2) = 8 \longrightarrow$ abs max @ $x=2$

$y(-1) = -1 \longrightarrow$ abs min @ $x=-1$



Ex: Find abs max/min for

$$f(x) = \frac{x^3}{3} - x \quad \text{over } [-3,4]$$

$f'(x) = x^2 - 1$

crit pts: $x = \pm 1$

Pts to check: $x = \pm 1$, $x=-3$, $x=4$

$f(-1) = 2/3$

$f(-3) = -6 \longrightarrow$ **abs min @ $x=-3$**

$f(1) = -2/3$

$f(4) = \frac{64}{3} - 4 = \frac{64}{3} - \frac{12}{3} = \frac{52}{3}$

$\rightarrow$ abs max @ $x = 4$.