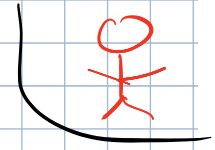


- HW on Sections 18 & 19
  due today 10/18

- Quiz on Sections 18 & 19
  Wed. 10/20

- Exam 2 10/27

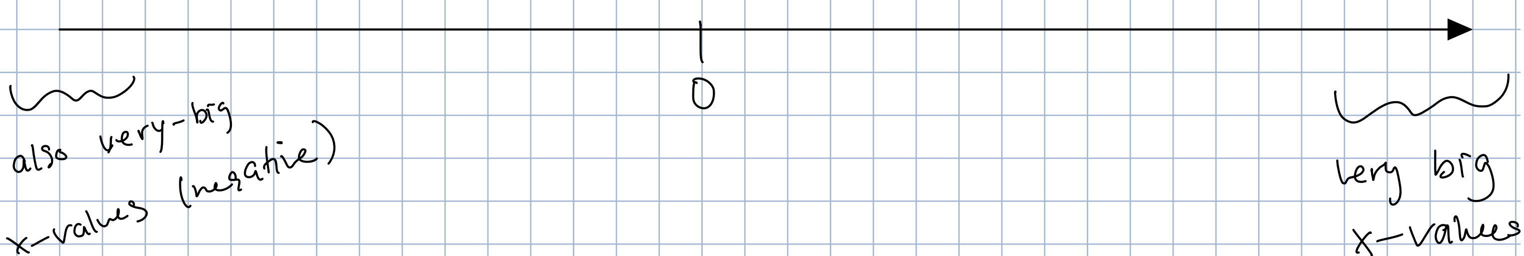
Section 20: Finding Asymptotes

(Horizontal
Vertical)

Horizontal: a way to describe the "tail-end" behavior of a function

Consider $y = \frac{1}{x}$

Goal: understand how y behaves at "tail-end" of the #-line



- For the right hand side behavior, we use the following notation

$$\lim_{x \to \infty} \frac{1}{x}$$

- For the left hand side behavior, we use the following notation

$$\lim_{x \to -\infty} \frac{1}{x}$$

Fact: If $c \in \mathbb{R}$ and $r > 0$. Then, if $c \neq 0$.

- $$\lim_{x \to \infty} \frac{c}{x^r} = 0$$

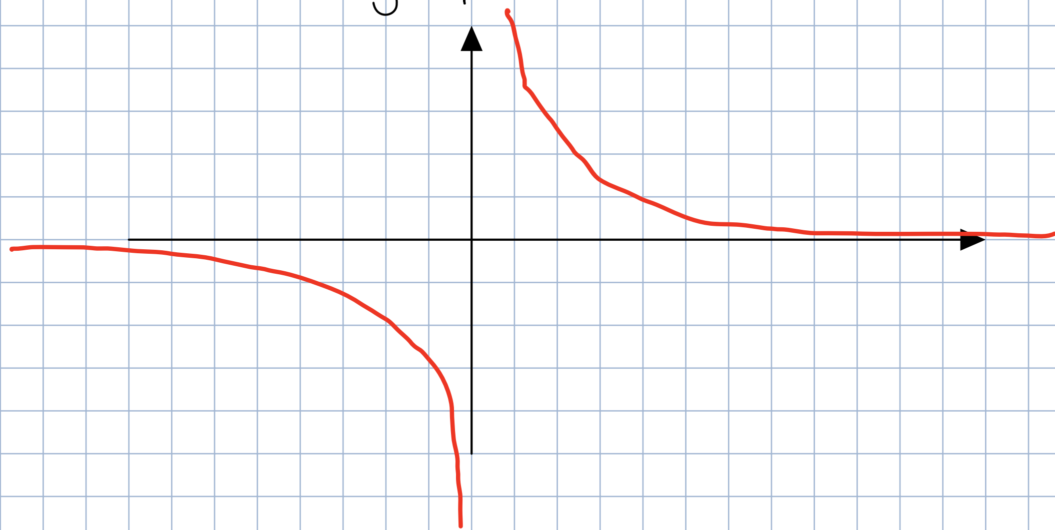
- $$\lim_{x \to -\infty} \frac{c}{x^r} = 0$$

Let's revisit $y = \frac{1}{x}$.

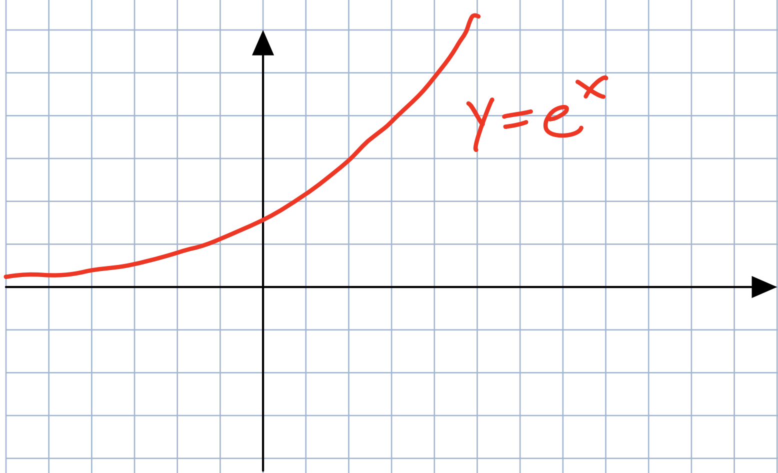
$$\lim_{x \to \infty} \frac{1}{x} \longrightarrow \frac{1}{\infty} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x} \longrightarrow \frac{1}{\infty} = 0$$

Since we get a #, our function has a horiz. asymptote



* Warning: Not all functions exhibit same LH / RH behavior



$$\lim_{x \to \infty} e^x = \infty$$

$$\lim_{x \to -\infty} e^x = 0$$

Ex: Evaluate $\displaystyle\lim_{x \to \infty} \frac{3x+1}{5x}$.

- Choose largest power in denom: x^1
- Divide everything by

$$\lim_{x\to\infty} \frac{3x+1}{5x} = \lim_{x\to\infty} \frac{\frac{3x}{x} + \frac{1}{x}}{\frac{5x}{x}}$$

$$= \lim_{x\to\infty} \frac{3+\frac{1}{x}}{5}$$

$$= \frac{3 + \cancel{\lim_{x\to\infty} \frac{1}{x}}}{5} = \frac{3+0}{5}$$

$$= \frac{3}{5}$$

So, f(x) has a horiz. asympt.
@ $y = 3/5$

Ex: Evaluate: $\lim_{x\to-\infty} \frac{6x^5+2x^2+x+1}{-2x^5-x^2+1}$

→ largest term in denom : x^5

$$\Rightarrow \lim_{x\to-\infty} \frac{\frac{6x^5}{x^5} + \frac{2x^2}{x^5} + \frac{x}{x^5} + \frac{1}{x^5}}{\frac{-2x^5}{x^5} - \frac{x^2}{x^5} + \frac{1}{x^5}}$$

$$= \lim_{x\to-\infty} \frac{6 + \cancel{\frac{2}{x^3}} + \cancel{\frac{1}{x^4}} + \cancel{\frac{1}{x^5}}}{}$$

$$-2 \quad - \cancel{\frac{1}{x^3}} \quad + \cancel{\frac{1}{x^5}}$$

$$= \frac{6}{-2} = -3$$

So $g(x)$ has a horiz asymp. @ $y=-3$

Vertical Asymptotes: A function will have a vert. asymp. at $x=a$ if:

- $\lim_{x \to a^-} f(x) = \pm\infty$

OR

- $\lim_{x \to a^+} f(x) = \pm\infty$

at $x=a$, f grows without bound

* Only one of these one-sided limits needs to be infinite in order to have a vert. asympt.

Algebraically: We'll rely on the following

$$\lim_{x \to a^\pm} f(x) \quad \frac{\text{"}c\text{"}}{0^\pm} = \pm\infty$$

$c \neq 0$

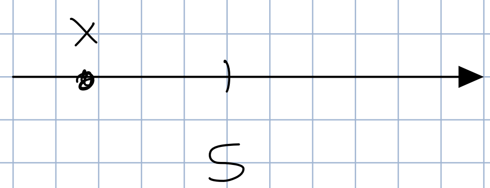
$\Rightarrow$ Look at places where denom $= 0$!

Ex: Find verti asymptotes for

$$f(x) = \frac{-3}{5-x}$$

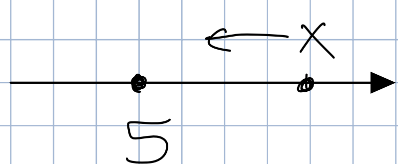
$\Rightarrow f$ is undefined at $x = 5$!

- $$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} \frac{-3}{5-x} \rightarrow \text{“}\frac{-3}{0^+}\text{”} = -\infty$$



$$x < 5 \Rightarrow 0 < 5 - x$$

- $$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} \frac{-3}{5-x} \rightarrow \text{“}\frac{-3}{0^-}\text{”} = \infty$$



$$5 < x$$

$$5 - x < 0$$

Ex: Find vertical asymp.:

a) $f(x) = \dfrac{x^2 - x - 6}{x^2 - 4}$

Note: $f(x)$ is undefined at $x = \pm 2$

Check for VA: $x = 2$, $x = -2$.

$\boxed{x = 2}$

$$\lim_{x \to 2^-} \frac{x^2 - x - 6}{x^2 - 4} = \lim_{x \to 2^-} \frac{(x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$= \lim_{x \to 2^-} \frac{x-3}{x-2} \begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \frac{-1}{0^-}$$

$$= \infty$$

$$\lim_{x \to 2^+} \frac{x^2 - x - 6}{x^2 - 4} = \lim_{x \to 2^+} \frac{(x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$= \lim_{x \to 2^+} \frac{(x-3)}{(x-2)} \begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \frac{\text{“}-1\text{”}}{0^+}$$

$$-\infty$$

$$\boxed{x = -2}$$

$$\lim_{x \to -2^-} \frac{x^2 - x - 6}{x^2 - 4} = \lim_{x \to -2^-} \frac{x-3}{x-2}$$

$$= \frac{-2-3}{-2-2} = 5/4$$

$$\lim_{x \to -2^+} \frac{x^2 - x - 6}{x^2 - 4} = \lim_{x \to -2^+} \frac{x-3}{x-2}$$

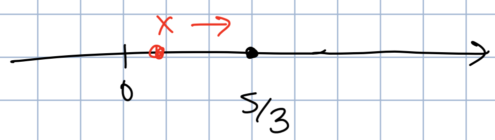
$$= 5/4$$

b) $g(x) = \dfrac{\sqrt{5x^2+1}}{3x-5}$

$\Rightarrow$ Check where denom $= 0$

$$\boxed{x = 5/3}$$

$$\lim_{x \to 5/3^-} \frac{\sqrt{5x^2+1}}{3x-5} \longrightarrow \frac{\text{"}C\text{"} > 0}{0^-} = -\infty$$

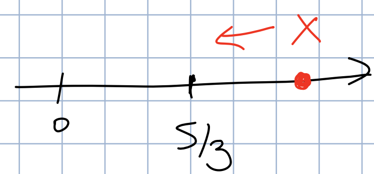


$x < 5/3$

$\Rightarrow \quad 3x < 5 \quad \Rightarrow \quad 3x - 5 < 0$

$$\lim_{x \to 5/3^+} \frac{\sqrt{5x^2+1}}{3x-5} \longrightarrow \frac{\text{"}C\text{"} > 0}{0^+} = \infty$$

$x \to 5/3^+$



$5/3 < x \implies 0 < 3x - 5$

Ex: Find all horiz. asympt.

a) $\dfrac{2x^3 + x - 4}{x^4 + x^3 + x}$

$$\lim_{x \to \infty} \frac{2x^3 + x - 4}{x^4 + x^3 + x} = \lim_{x \to \infty} \frac{\cancel{\frac{2}{x}} + \cancel{\frac{1}{x^3}} - \cancel{\frac{4}{x^4}}}{1 + \cancel{\frac{1}{x}} + \cancel{\frac{1}{x^3}}}$$

$$= \frac{0+0+0}{1+0+0} = 0$$

$$\lim_{x \to -\infty} \frac{2x^3 + x - 4}{x^4 + x^3 + x} = 0$$

So, f(x) has a horiz. asymp.

@ $y=0$

b) $\dfrac{2x^7 + x^2 - 12}{3x^4 + 1}$

$$\lim_{x \to \infty} \frac{2x^7 + x^2 - 12}{3x^4 + 1} = \lim_{x \to \infty} \frac{2x^3 + \cancel{\frac{1}{x^2}} - \cancel{\frac{12}{x^4}}}{3 + \cancel{\frac{1}{x^4}}}$$

$$= \infty$$

$$\lim_{x \to -\infty} \frac{2x^7 + x^2 - 12}{3x^4 + 1} = \lim_{x \to -\infty} \frac{2x^3 + \cancel{\frac{1}{x^2}} - \cancel{\frac{12}{x^4}}}{3 + \cancel{\frac{1}{x^4}}}$$

$$= -\infty$$

So, this fct has no horiz. asympt.

c) $\dfrac{\sqrt{5x^2+1}}{3x-5}$

$$\lim_{x\to\infty} \frac{\sqrt{5x^2+1}}{3x-5} = \lim_{x\to\infty} \frac{\left(\frac{\sqrt{5x^2+1}}{x}\right)}{\left(3-\frac{5}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\frac{\sqrt{5x^2+1}}{\sqrt{x^2}}}{\left(3-\frac{5}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{5x^2+1}{x^2}}}{\left(3-\frac{5}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{5+\cancel{\frac{1}{x^2}}}}{\left(3-\cancel{\frac{5}{x}}\right)}$$

$$= \sqrt{5}/3$$

Finish other limit on Wed.