



• HW due on today, Section 15 & 17
(By end of day)

• Quiz 5 on Wed. (Sections 15 & 17)

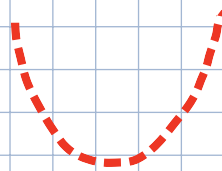
• Exam 2 \rightarrow Oct 27th

Section 18: Concavity, SDT.

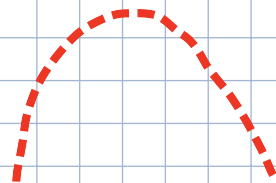
Recall:

Concavity \rightarrow way in which a function "curves"

$f''(x) > 0 \Rightarrow$ Concave up



$f''(x) < 0 \Rightarrow$ Concave down



EX:

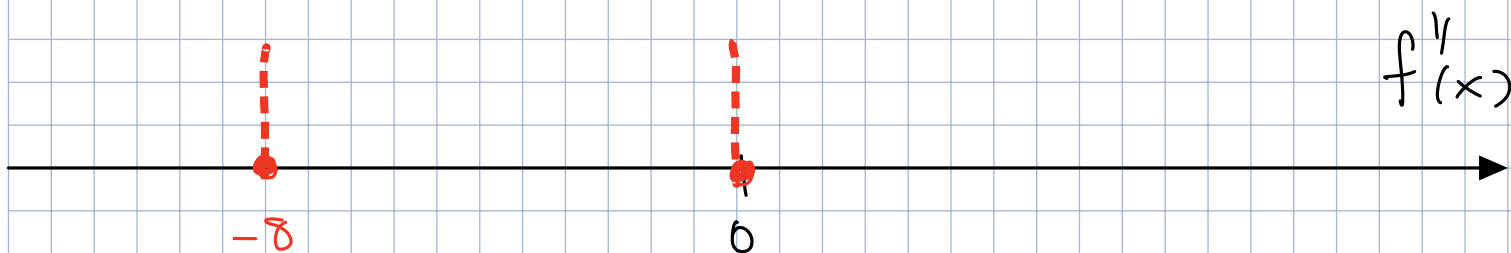
Find intervals of concavity:
and points of inflection

$$f(x) = \frac{1}{2}x^2 + \frac{9}{2}x^{5/3} + x - 7$$

$$f'(x) = x + \cancel{\left(\frac{5}{3}\right)} \left(\cancel{\frac{9}{5}}\right)^3 x^{2/3} + 1$$

$$f'(x) = x + 3x^{2/3} + 1$$

$$\Rightarrow f''(x) = 1 + 2x^{-1/3}$$



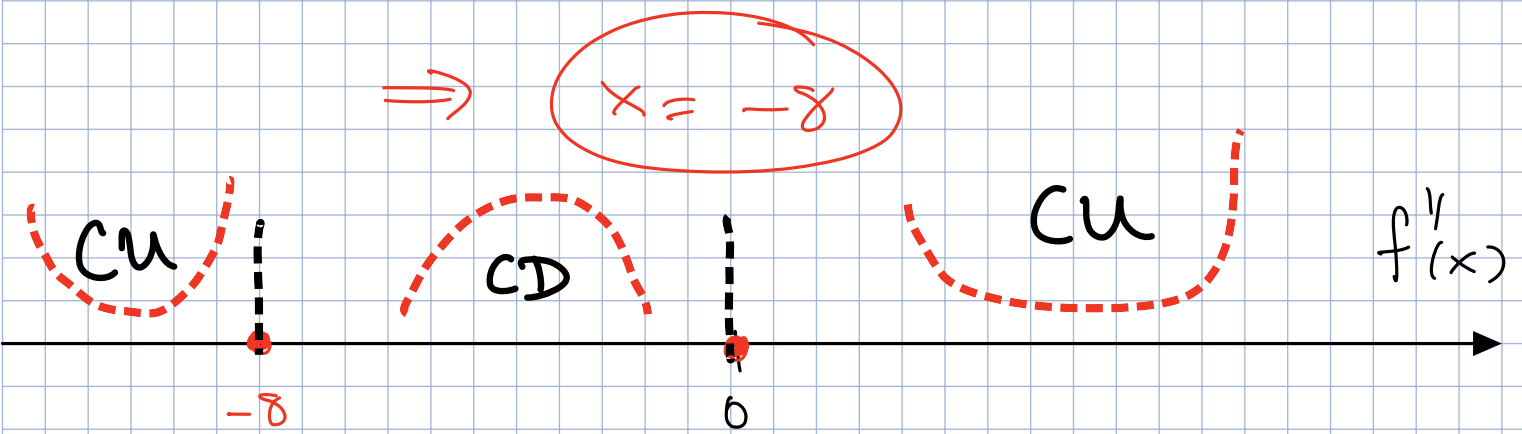
$f''(x)$ und ? $x=0$

$$f''(x) = 0 ?$$

$$f''(x) = 0 = 1 + \frac{2}{x^{1/3}}$$

$$\Rightarrow \frac{x^{1/3} + 2}{x^{1/3}} = 0$$

$$\Rightarrow x^{1/3} = -2$$



$$f''(-27) = 1 + \frac{2}{(-27)^{1/3}}$$

$$1 - \frac{2}{3} = \frac{1}{3} > 0$$

$$f''(-1) = 1 + \frac{2}{(-1)} = -1 < 0$$

$$f''(8) = 1 + \frac{2}{2} = 2 > 0$$

So, f is concave up on $(-\infty, -8) \cup (0, \infty)$

f is concave down on $(-8, 0)$

lastly, f has inf. pts at $x=0, x=-8$

Ex.

Find intervals of concavity
and pts of inf.

$$f(x) = xe^x.$$

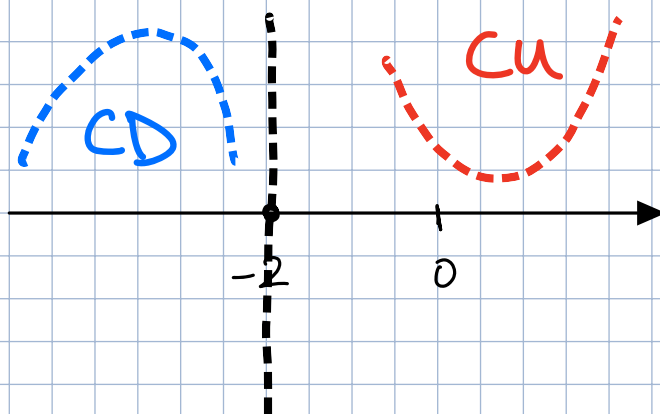
$$f'(x) = xe^x + e^x$$

$$f''(x) = xe^x + e^x + e^x$$

$$f''(x) = e^x(2+x)$$

$$\frac{f''(x) \text{ und?}}{\text{Never!}}$$

$$\frac{f''(x) = 0?}{\Rightarrow x = -2}$$



f is CU on $(-2, \infty)$

f is CD on $(-\infty, -2)$

f has an inf pt. @ $x = -2$

Ex:

Find all local extrema
of $f(x) = x^3 + 2x^2 + x + 6$
(Using SDT)

$$\Rightarrow f'(x) = 3x^2 + 4x + 1$$

$$\Rightarrow f''(x) = 6x + 4$$

→ Find crit pts:

$$f'(x) = 3x^2 + 4x + 1$$

$$(3x+1)(x+1)$$

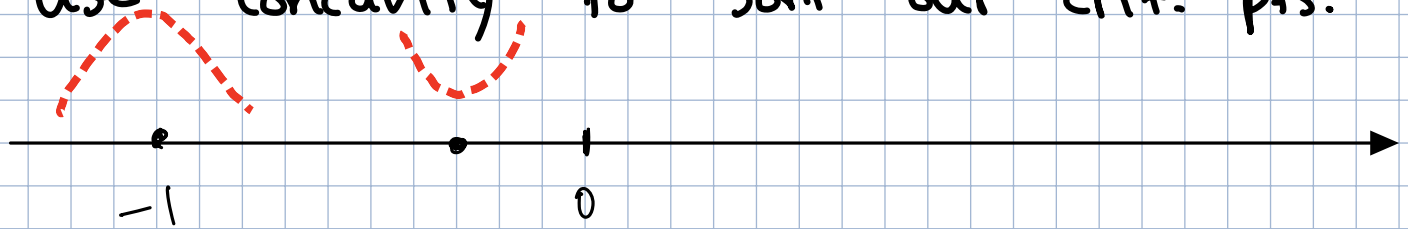
$f'(x)$ und?

Never!

$f'(x) = 0$?

$$x = -\frac{1}{3}, x = -1$$

Use concavity to sort our crit. pts.



$$\begin{aligned} f''(-1/3) &= 6(-1/3) + 4 \\ &= -2 + 4 = 2 > 0 \end{aligned}$$

So f has a local min @ $x = -1/3$

$$\begin{aligned} f''(-1) &= 6(-1) + 4 \\ &= -2 < 0 \end{aligned}$$

So f has a local max @ $x = -1$

Ex:

Find all extrema for

$$f(x) = x - \ln(x)$$

(use SDT)

$$f'(x) = 1 - \frac{1}{x}$$

→ $f'(x)$ is und.?

$x=0$ → Not a crit pt
b/c it's not in domain!

→ $f'(x) = 0$?

$$x=1$$

$$f''(x) = \frac{1}{x^2}$$

What is concavity @ $x=1$?

$$f''(1) = \frac{1}{1} > 0$$

So by SDT, f has a local min
@ $x=1$

Section 19: Graphing Functions
(w/o asymptotes)

Steps:

1. Find domain

2. ^{*} Find auxiliary info (Roots, y-intercept)

3. Find critical pts

4. Find intervals of inc./dec

5. Find intervals of concavity

6. Sketch graph

EX: Sketch $y = x^4 - 4x^3 + 10$
on $[-1, 4]$

Domain: $[-1, 4]$

Roots: $x^4 - 4x^3 + 10 = 0$

→ too ugly to solve

y-intercept:

$$f(0) = 10 \rightarrow (0, 10)$$

Crit. pts: $f'(x) = 4x^3 - 12x^2$

$f'(x)$ und ?

N.A.

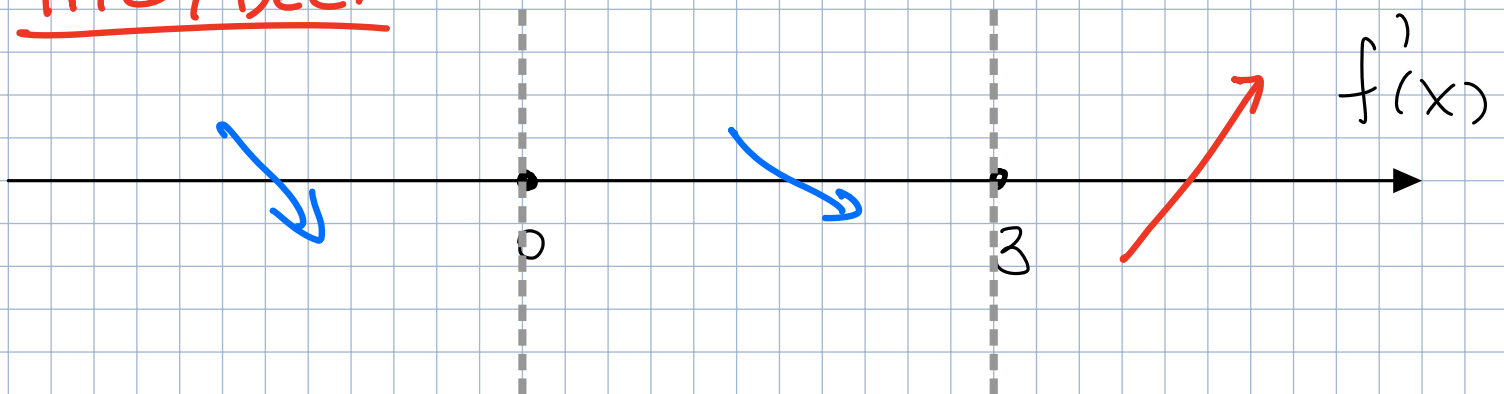
$$f'(x) = 0 ?$$

$$4x^2(x-3) = 0$$

$$x=0$$

$$x=3$$

Inc/Dec:



Concavity:

$$f''(x) = 12x^2 - 24x$$

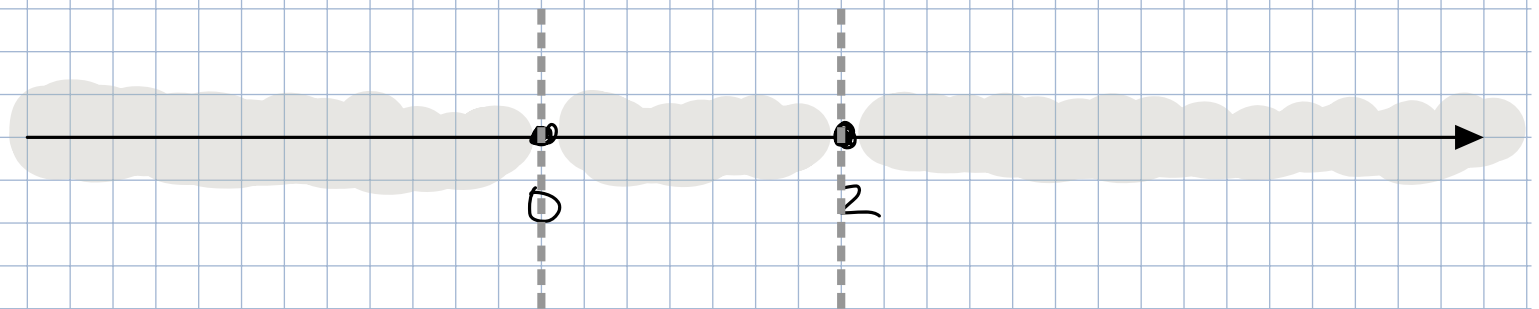
f'' is und ?

N.A.

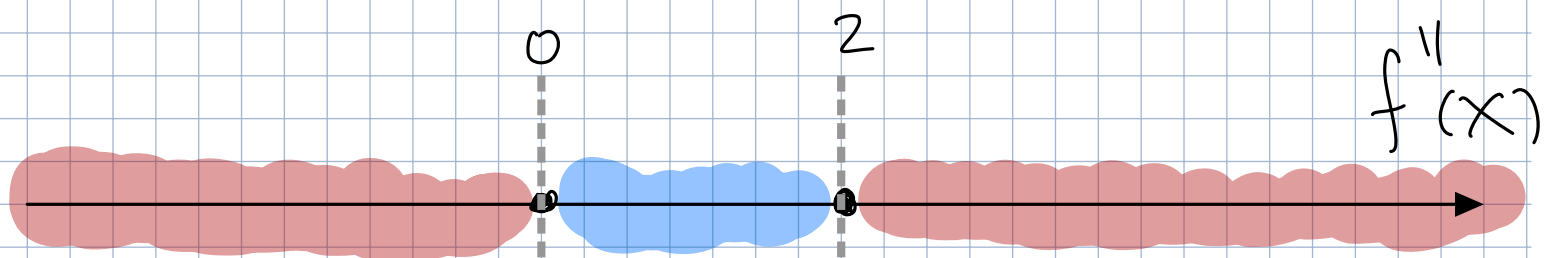
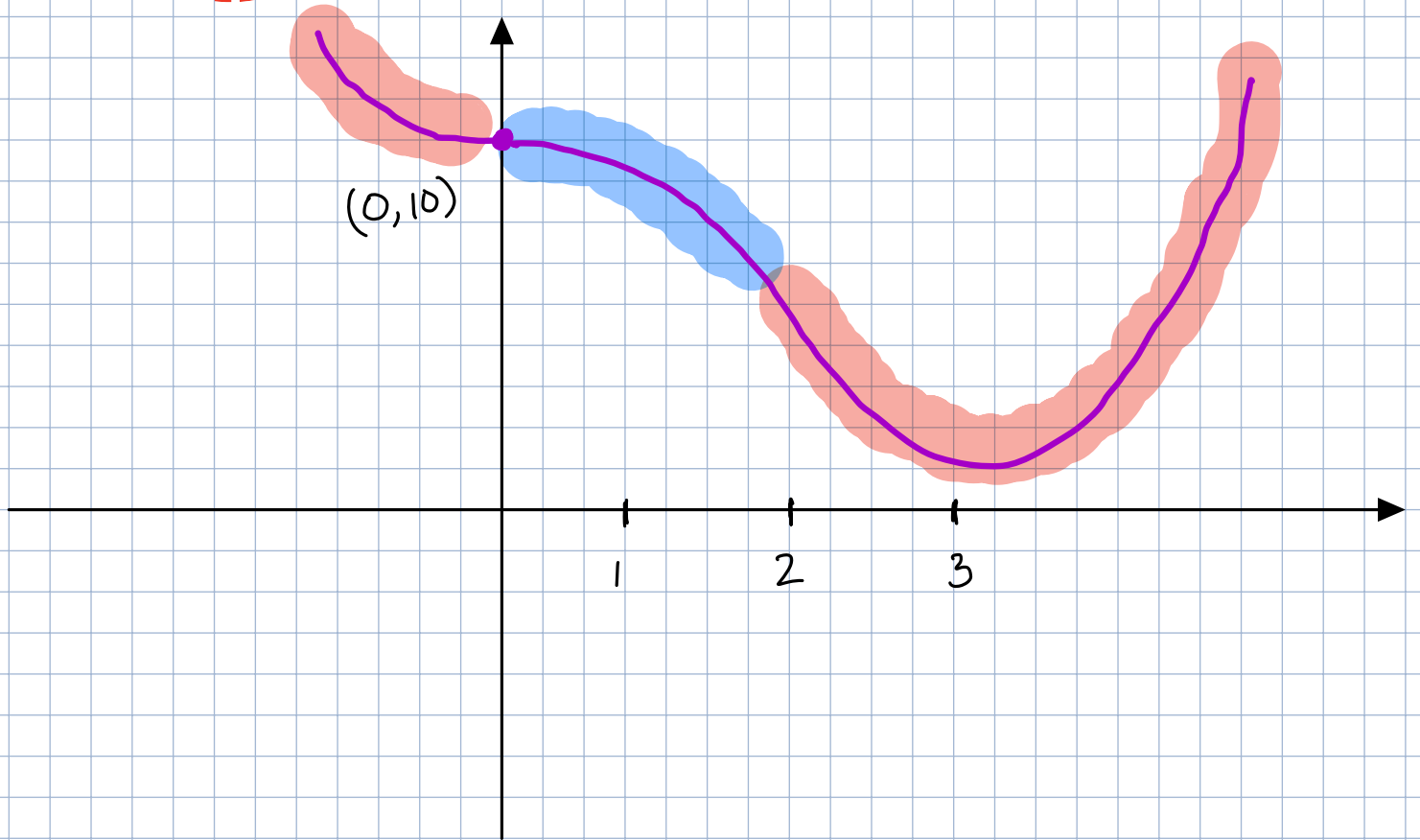
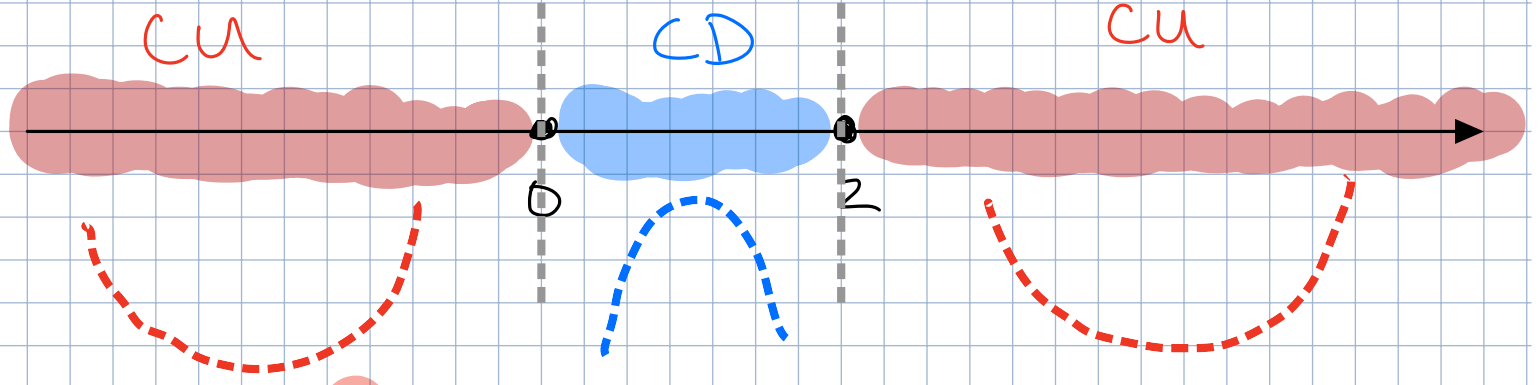
$$f''(x) = 0 ?$$

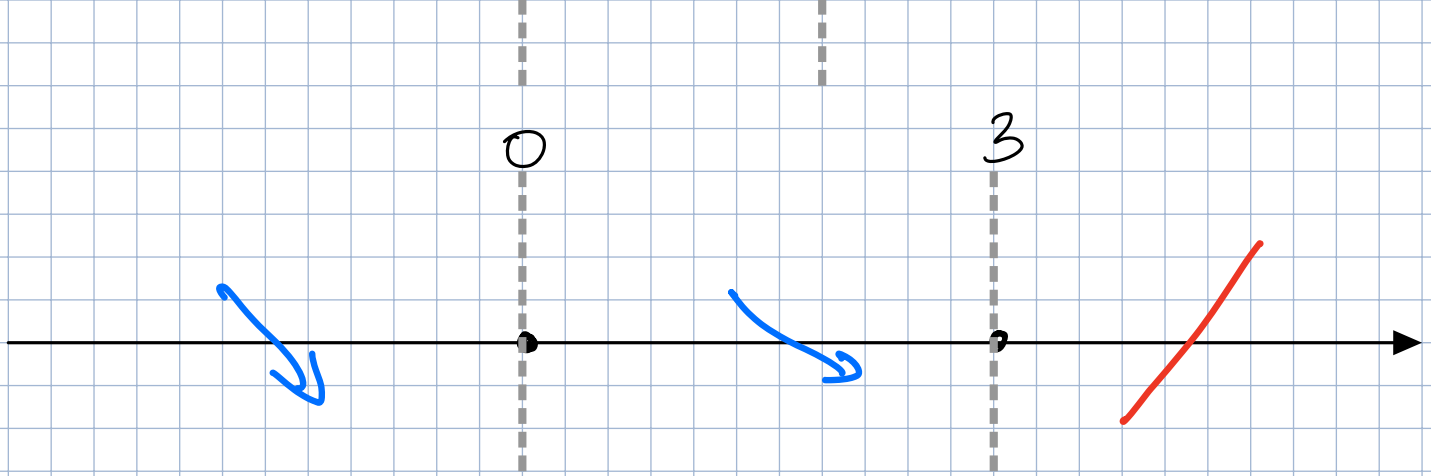
$$x=0$$

$$x=2$$



↓ after sample pts ↓





Ex:

Sketch $f(x) = \frac{1}{2}x - \sqrt{x}$

Domain: $[0, \infty)$

Roots: $f(x) = \frac{1}{2}x - \sqrt{x} = 0$

$$\Rightarrow \sqrt{x} \left(\frac{1}{2}\sqrt{x} - 1 \right) = 0$$

$x=0$	$x=4$

$(0, 0)$, $(4, 0)$

Crit. Pts: $f'(x) = \frac{1}{2} - \frac{1}{2}x^{-1/2}$

$$= \frac{1}{2} \left(1 - \frac{1}{\sqrt{x}} \right)$$

$$f'(x) = 0 \quad ?$$

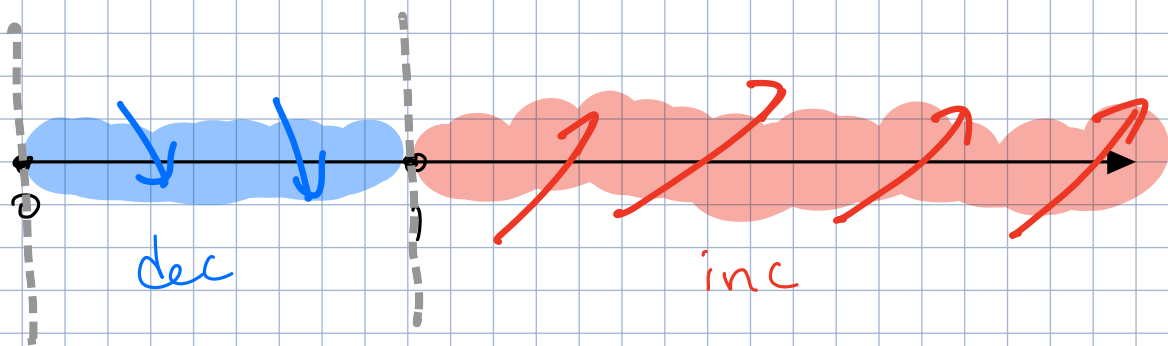
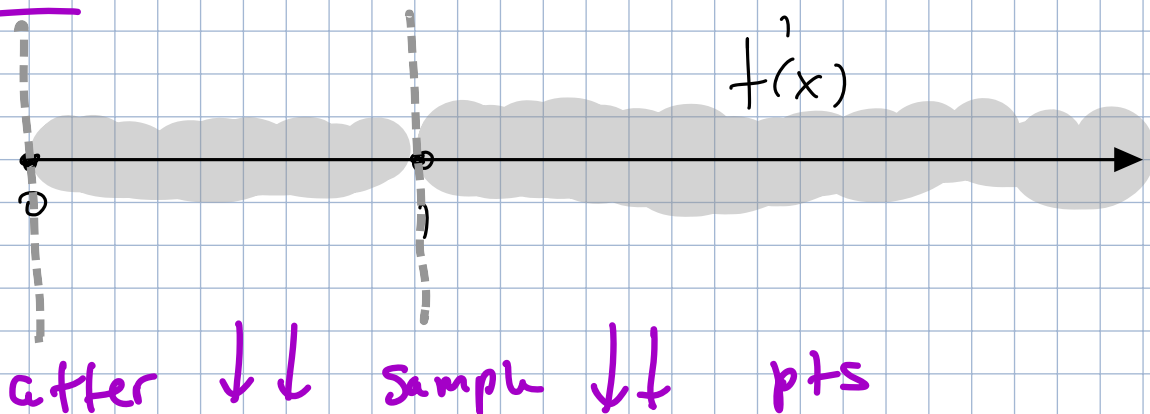
$f(x)$ is und.?

$$\frac{1}{2} = \frac{1}{2} \frac{1}{\sqrt{x}}$$

f is und @ $x=0$

$$x=1$$

Inc / Dec:



Concavity:

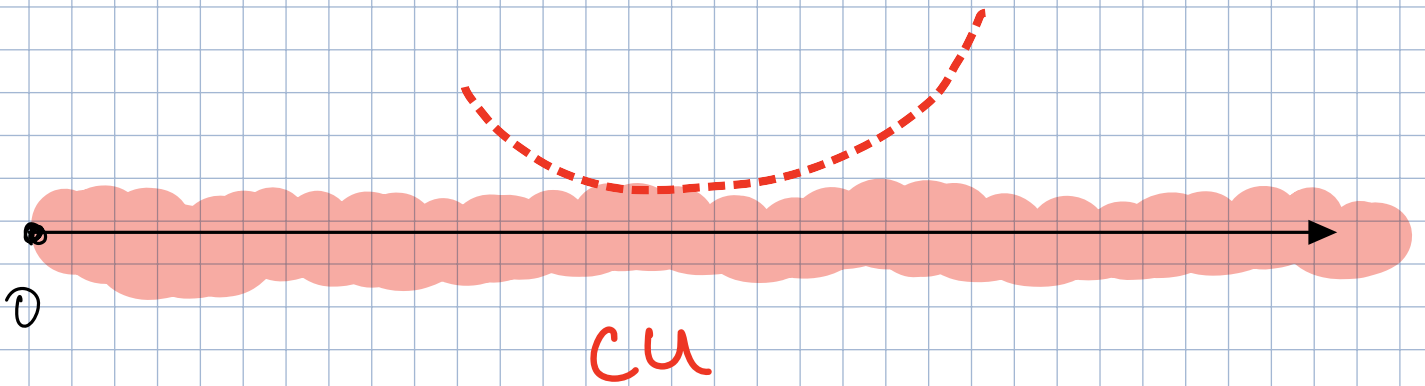
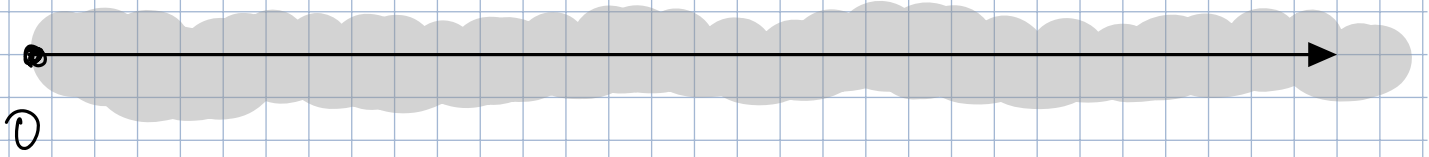
$$\begin{aligned} f''(x) &= \frac{1}{4} x^{-3/2} \\ &= \frac{1}{4 x^{3/2}} \end{aligned}$$

$$f''(x) = 0 ?$$

N.A.

$$f''(x) \text{ und?}$$

$$x = 0$$



CU

