- HW due on Monday (Section 15 & 17) by end of day

- Quiz 5 on Wed. (Sections 15 & 17)

- Exam 2 $\longrightarrow$ Oct 27^{th}

Ex:

Let $f(x) = \dfrac{x^4}{4} - \dfrac{x^3}{3} - 3x^2$

a) Locate and classify all extrema

b) Find intervals of inc/dec.

a) Find crit pts first!

$$f'(x) = x^3 - x^2 - 6x$$

$\rightarrow$ $f'(x)$ is und?

Never!

$\rightarrow$ $f'(x) = 0$?

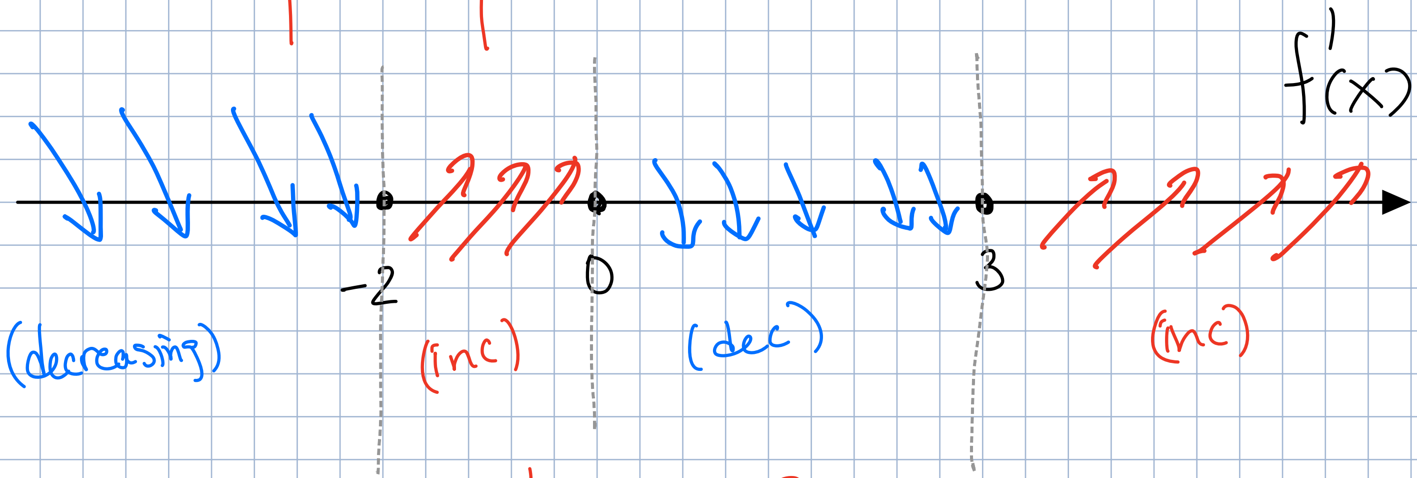
$$x^3 - x^2 - 6x = 0$$

$$x(x^2 - x - 6) = 0$$

$$x(x-3)(x+2) = 0$$

$x(x-3)(x+2)=0$

$x=0$	$x=3$	$x=-2$



$$f'(x) = x^3 - x^2 - 6x$$
$$= x(x-3)(x+2)$$

$f'(-10) < 0$

$f'(-1) > 0$

$f(1) < 0$

$f(10) > 0$

So @ $x=-2$, f has a local min

$x=0$, f has a local max

$x=3$, f has a local min.

b) f is increasing on

$(-2,0)\cup(3,\infty)$

f is decreasing on

$(-\infty,-2)\cup(0,3)$

Section 18: 2^{nd} derivative test

→ Another tool for classifying extrema.

Let $f(x)$ be twice differentiable.

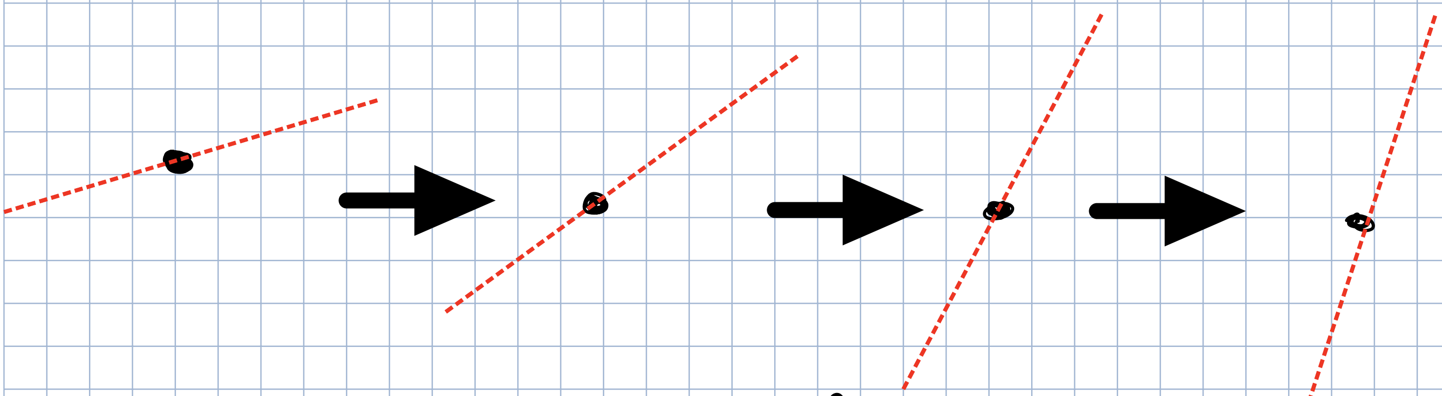
Recall: $f''(x) = \frac{d^2}{dx^2} f(x)$ → 2^{nd} deriv

$\frac{d}{dx}\left[f'(x)\right]$

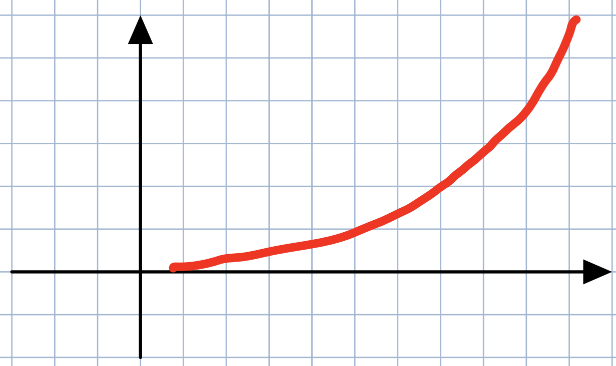
$g'(x)$ → rate of change of g.

$\frac{d}{dx}[f'(x)] \to$ rate of change of f' i.e. the slope.

· Assume $f''(x) > 0$: So $f'(x)$ is increasing

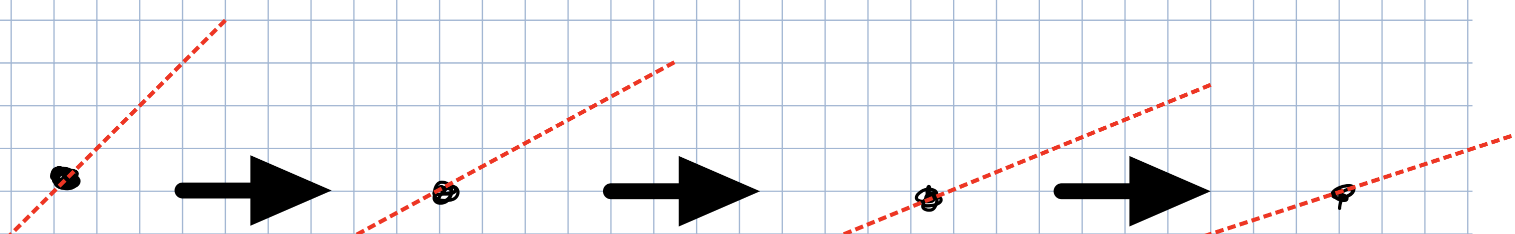


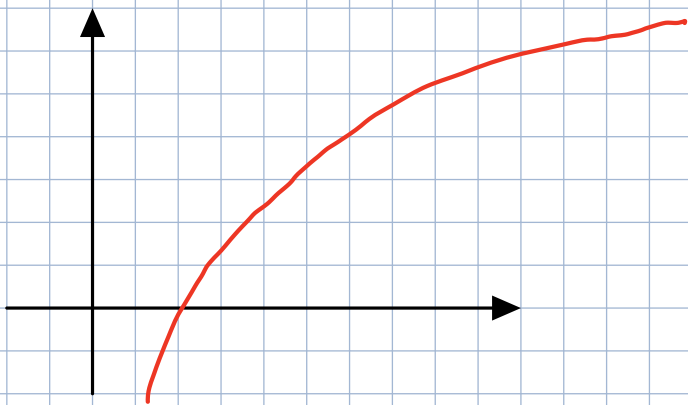
This creates the following shape



$\Longrightarrow$ Concave up

· Assume $f''(x) < 0$: $f'(x)$ is decreasing





$\Rightarrow$ Concave down

<u>Def^n</u>:

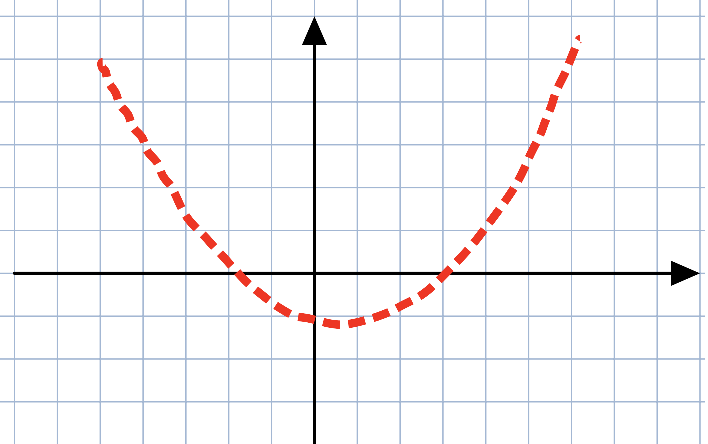
Let f be twice diffble. We say that

- f is concave up if $f''(x) > 0$

- f is concave down if $f''(x) < 0$

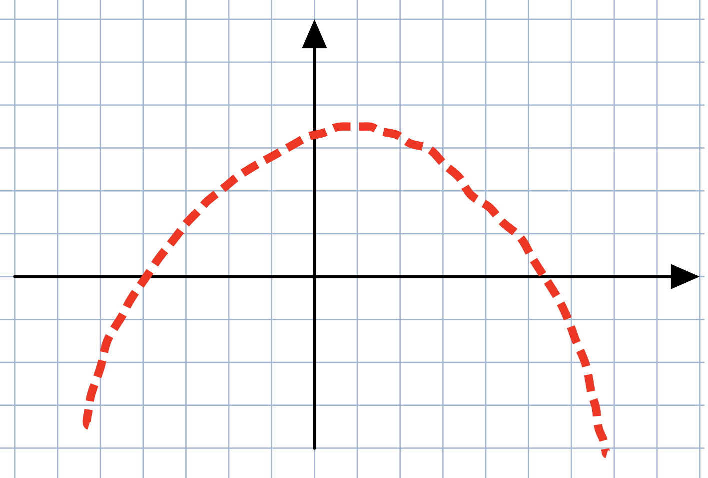
"Concave up"

$f''(x) > 0$

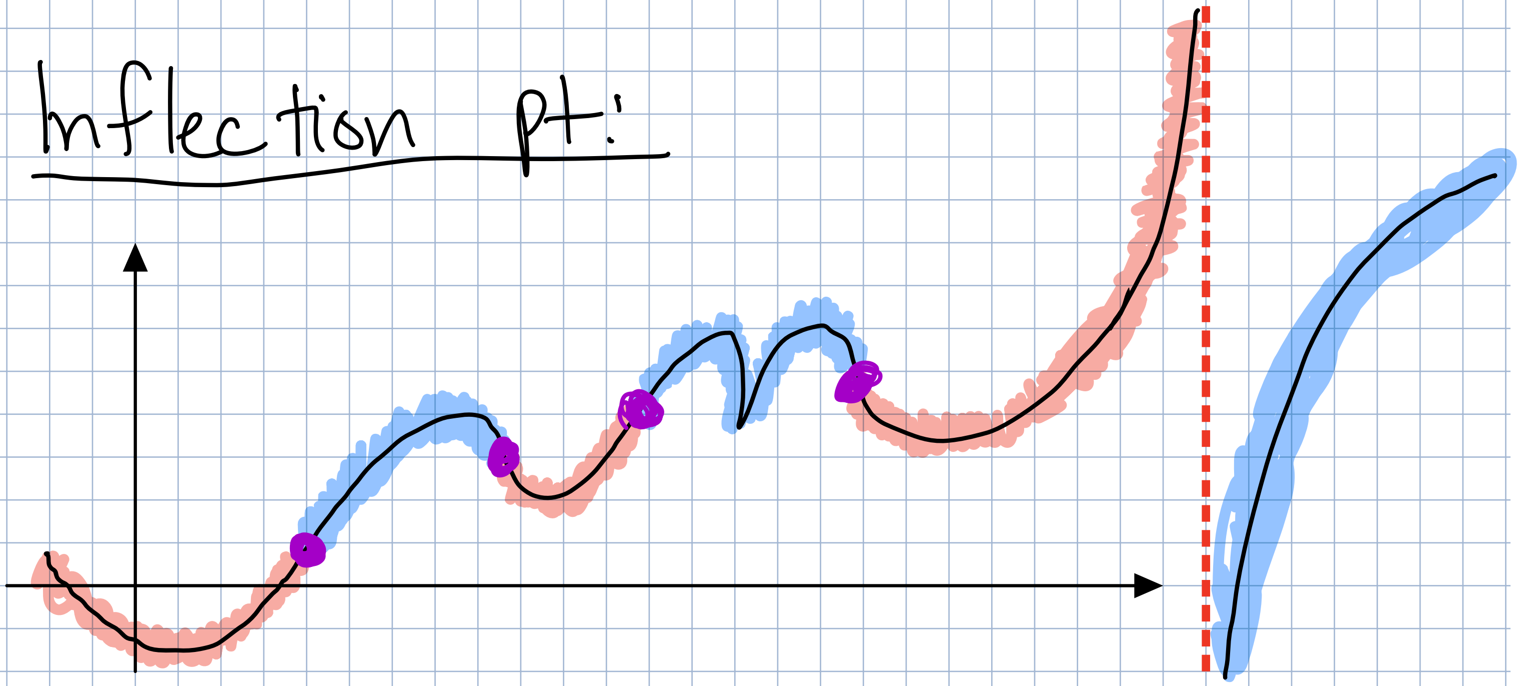


"Concave down"

$f''(x) < 0$



Inflection pt:



Red $\Rightarrow$ Concave up   CU

blue $\Rightarrow$ Concave down   CD

Inflection pt $\Rightarrow$ pts where f changes concavity

• Notice that if $x=a$ is a crit pt with $f''(a) > 0$ $\Rightarrow$ $x=a$ is a local min

• Notice that if $x=a$ is a crit pt, with $f''(a) < 0$ $\Rightarrow$ $x=a$ is a local max

Ex: Find intervals of concavity for

$$f(x)=\frac{x^5}{5}-x^4-5x$$

$$f'(x)=x^4-4x^3-5$$

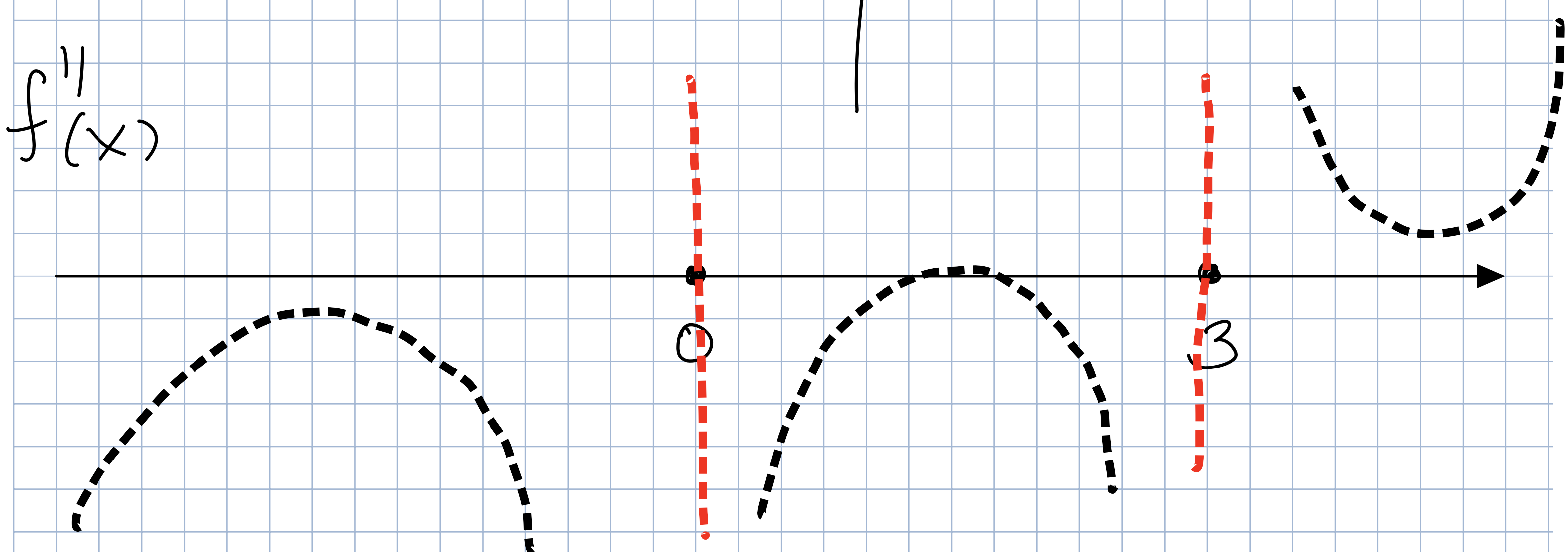
$$f''(x)=4x^3-12x^2$$

→ $f''(x)$ und? Never!

→ $f''(x)=0$? $4x^3-12x^2=0$

$$4x^2(x-3)=0$$

$x=0$	$x=3$



$f''(x) = 4x^2(x-3)$

$f''(-1) < 0$

$f''(1) < 0$

$f''(10) > 0$

f is CU on $(3, \infty)$

f is CD on

$(-\infty, 0) \cup (0, 3)$