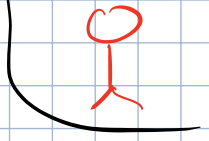


- Quiz 4 (Oct 8th)
  → Sections 13 & 14

- HW on sections 15 & 16 will be due Oct 11th

- Quiz 5 on Sections 15 & 16 (Oct 13th)
  → Sections 15 & 16

- Exam 2 scheduled for Oct 27th
  → Sections 13 → 22

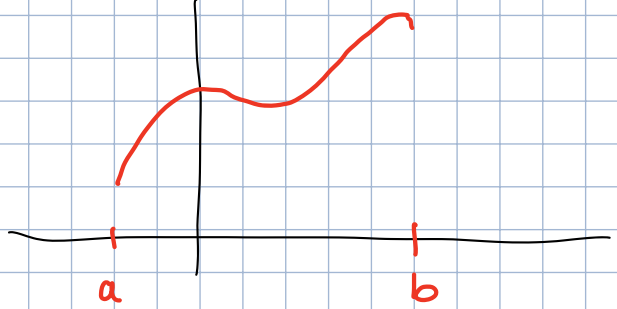
Section 16 (very brief).

→ Nice theorems.

Theorem (Mean Value Theorem)

Let $f(x)$ be cont. on $[a,b]$ and differentiable on (a,b). Then, there exists a pt $x_0 \in (a,b)$ so that

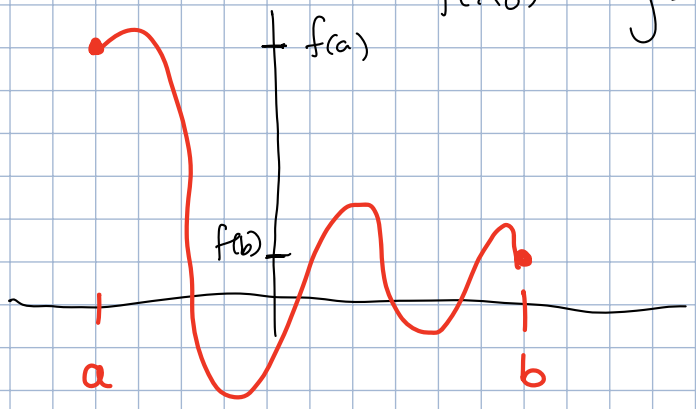
$$f'(x_0) = \frac{f(b) - f(a)}{b-a}$$



Theorem (Intermediate Value theorem)

Let f be cts on $[a,b]$ suppose $f(a) \neq f(b)$. Then if y is some value between $f(a)$ and $f(b)$, there exists a pt $x_0 \in (a,b)$ so that

$$f(x_0) = y.$$



Theorem: (Extreme Value theorem)

Let f be a cts function on $[a,b]$.

Then there are numbers m and M in $[a,b]$ so that

$$f(m) \leq f(x) \leq f(M) \quad \text{for all } x \in [a,b]$$

Section 17: (First Deriv. Test)

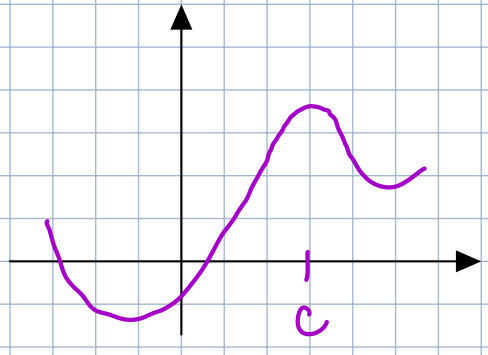
→ sort crit. pts into
- maximum
- minimum
- neither

Recall:

A function is decreasing if $f'(x) < 0$

A function is increasing if $f'(x) > 0$

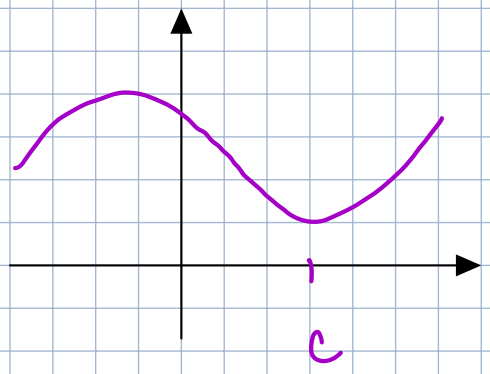
Suppose $c \in \mathbb{R}$ is a local max for $f(x)$:



- To the left of $x=c$, our function is increasing
  $f'(x) > 0$

- To the right of $x=c$, our function is decreasing
  $f'(x) < 0$

Suppose $c \in \mathbb{R}$ is a local **min** for $f(x)$:



- To the left of $x=c$, our function is decreasing

$$f'(x) < 0$$

- To the right of $x=c$, our function is increasing

$$f'(x) > 0$$

Ex: Identify local extrema.

$$f(x) = \frac{3x^2}{2} - 7x + 2$$

First: find all crit pts!

$$f'(x) = 3x - 7$$

- $f'(x) = 0$?

$$f'(x) = 3x - 7 = 0$$

$$\Rightarrow x = 7/3$$

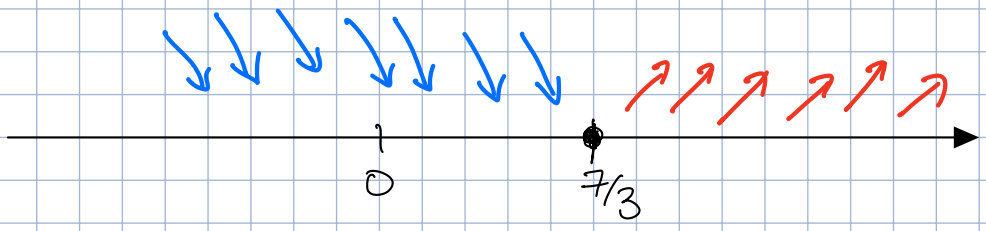
← our only crit pt

- $f'(x)$ is und?

$$f'(x) = 3x - 7$$

$f'(x)$ is always defined!

$\Rightarrow$ Use the FDT to classify $x = 7/3$:



Observe behavior of $f'(x)$ around our pt!

Sample pt:

$x = 0 \Rightarrow f'(0) = -7$ our function is decreasing

Sample pt:

$x = 10 \Rightarrow f'(10) = 23 > 0$ our function is inc.

By the FDT, $x = 7/3$ is a local min!

Ex: Find all local extrema (if any)

$$f(x) = \frac{1}{x}$$

Find all crit. pts!

$$f'(x) = \frac{-1}{x^2}$$

- $f'(x)$ is und? $x = 0$
- $f'(x) = 0$? no sol'ns

$x=0$ is not a crit. pt b/c $x=0$ is not in domain of $f(x)$.

Has no crit. pts! $\Rightarrow$ Has no local extrema.

Ex: Find all local extrema

$$f(x) = \frac{x^2 - 2x + 1}{x-3} = \frac{(x-1)^2}{x-3}$$

Find all crit. pts!

$$f'(x) = \frac{2(x-1)(x-3) - (x-1)^2}{?}$$

$$(x-3)^2$$

$$= \frac{2\left[x^2-4x+3\right]-\left(x^2-2x+1\right)}{(x-3)^2}$$

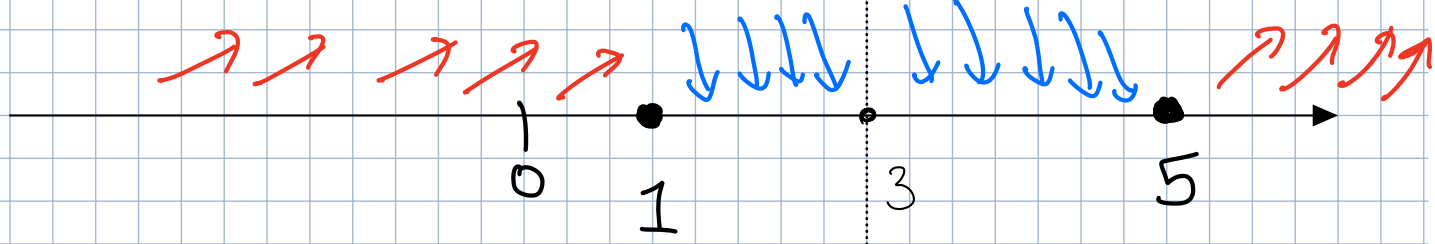
$$= \frac{2x^2-8x+6-x^2+2x-1}{(x-3)^2}$$

$$= \frac{x^2-6x+5}{(x-3)^2}$$

$$f'(x) = \frac{(x-5)(x-1)}{(x-3)^2}$$

$f'(x) = 0$? $\quad x=5 \quad x=1$

$f'(x)$ is und? $\quad x \neq 3$ not in domain of f'.



$f'(0) > 0$

$f'(2) < 0$

$f'(4) < 0$

$f'(20) > 0$

$x = 1$ is a local max

$x = 5$ is a local min