- Quiz on Friday Oct. 8^{th}
  (Sections 13, 14)

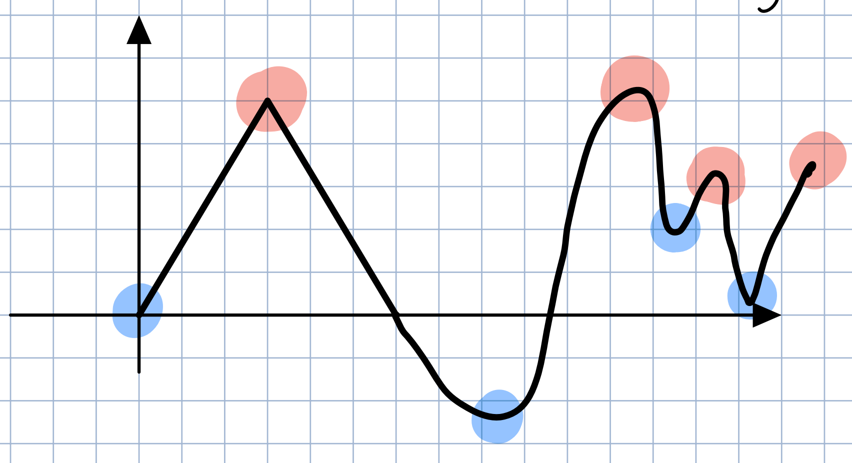
Section 15 : Local maxima / minima

Def^n (Relative / Local Extrema)

Let f be a real-valued function. We say that f has a relative maximum (or minimum) at $x=a$ if

- $f(x) \leq f(a)$, for all x "near" a.
- $f(x) \geq f(a)$, for all x "near" a.

Examples: Consider the following graph of $f(x)$:



$\rightarrow$ the red regions corresp. to local max

$\rightarrow$ the blue regions corresp. to local min

What do these spots have in common?

FACT: Let $f(x)$ be a real-valued function.

If $x=a$ is a relative extrema, then

- $f'(a)=0$, <u>OR</u>
- $f'(a)$ is undefined

<u>**Def^n:**</u> (Critical Point)

Let f be a real-valued fct. We say that a point $x=a$ in the domain is a <u>critical point</u> if

- $f'(a)=0$
- $f'(a)$ is undefined.

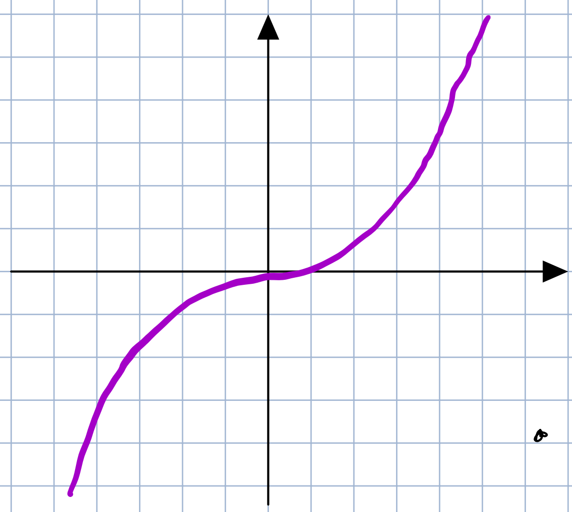
→ If we're looking for extrema, we should investigate the critical pts.!

* I like to think of crit. pts as "candidates for extrema".

Warning: A crit pt need not be a

local extremum.

Ex: Consider $f(x) = x^3$



$f'(x) = 3x^2$

Crit pts:

- $f'(x)$ und? never!
- $f'(x) = 0$? $x=0$

$x=0$ is a crit pt!

Ex: Find all crit pts

$$f(x) = 6x^3 - 3x^2 - 12x - 4$$

$\rightarrow f'(x) = 18x^2 - 6x - 12$

- $f'(x)$ undefined?

never!

$\cdot\ f'(x) = 0\ ?$

$$18x^2 - 6x - 12 = 0$$

$$\Rightarrow 6(3x^2 - x - 2) = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4(3)(-2)}}{6}$$

$$= \frac{1 \pm \sqrt{1+24}}{6} = \frac{1 + 5}{6}$$

$x = 1$ $\quad$ $x = -2/3$

Ex: Find all crit pts:

$$f(x) = \sqrt[3]{x^2 - x} = (x^2 - x)^{1/3}$$

$$f'(x) = \frac{1}{3}(x^2 - x)^{-2/3}(2x - 1)$$

$$f'(x) = \frac{2x - 1}{3\sqrt[3]{(x^2 - x)^2}}$$

• $f'(x)$ und ?

$$\Rightarrow f'(x) = \frac{2x-1}{3\sqrt[3]{(x^2-x)^2}}$$

$$\Rightarrow 3\sqrt[3]{(x^2-x)^2} = 0$$

$$\Rightarrow x^2 - x = 0$$

$$\Rightarrow x=0, \quad x=1$$

• $f'(x) = 0$?

$$\Rightarrow f'(x) = \frac{2x-1}{3\sqrt[3]{(x^2-x)^2}} = 0$$

$$\Rightarrow 2x-1 = 0$$

$$\Rightarrow x = 1/2$$

Ex: Find all crit pts

$$f(x) = x^{1/3} - x^{-2/3} = \sqrt[3]{x} - \frac{1}{\sqrt[3]{x^2}}$$

$$f'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3}$$

$$= \frac{1}{3} x^{-5/3} (x + 2)$$

- $f'(x)$ und ?

$$f'(x) = \frac{x+2}{3\,x^{5/3}} \implies x \neq 0$$

- $f'(x) = 0$?

$$f'(x) = \frac{x+2}{3\,x^{5/3}} = 0 \; ?$$

$$\implies x+2 = 0$$

$$\implies x = -2$$

Our <u>only</u> crit pt is $x = -2$.

($x=0$ is not in the domain of $f(x)$!)

Ex: Find all crit pts of

$$f(x) = 2^x \cdot x \qquad D_f = \mathbb{R}$$

$$f'(x) = \frac{d}{dx}\left[2^x\right]x + 2^x \cdot \frac{d}{dx}[x]$$

$$= \frac{d}{dx}\left[e^{x\ln(2)}\right]x + 2^x \cdot 1$$

$$= e^{x\cdot\ln(2)} \cdot \ln(2)\, x + 2^x$$

$$f'(x) = 2^x \cdot \ln(2)\cdot x + 2^x$$

$f'(x)$ und ?

$$\Rightarrow \quad f'(x) = x\ln(2)\cdot 2^x + 2^x$$

$$= 2^x\left(\ln(2)\cdot x + 1\right)$$

never!

$f'(x) = 0$?

$$\Rightarrow \quad f'(x) = 2^x\left(\ln(2)x + 1\right)$$

$2^x = 0$

no sol'ns

$\ln(2)x + 1 = 0$

$\Longrightarrow x = \frac{-1}{\ln(2)}$

$$(x^3)^{1/2} = x^{3/2}$$

$$x^{(3^{1/2})}$$

$$e^{x^2} \rightarrow e^{x^2} \cdot 2x$$