# Section 14: (cont'd)

## → Steps:

1.* Draw a picture (if possible)

2. Label all variables/quantities that we'll use.

3. Identify the problem's unknowns (goal) as well as the problem's known-valus/variables

4. Write down/come up with the "relevant eq$^n$".
   This is the eq$^n$ that relates our variables
   → Depends on problem!
   → may be volume eq$^n$, area eq$^n$, or some other eq$^n$

5. Differentiate the relevant eq$^n$ so that we can relate our derivatives (rates)
   → Often, we'll seek a rate that changes over time. So, we'll commonly use
   $$\frac{d}{dt}(\text{stuff})$$
   but it depends on the problem!

6. Use the derivative eq$^n$ (from step 5) and the known-valus/variables to solve for unknowns (goal)

⇒ Practice by using these steps!

**Example 14.1.** The profit, in dollars, that a company makes on one $q$ units of product is given by the equation $P(q) = 5q - 0.01q^2 - 1000$. The demand for the product is increasing at a rate of 10 units per week. How fast is the profit changing when the demand is at 100 units [redacted]?

$q$ = # of items

$P$ = profit

**Known:** $\frac{dq}{dt} = 10$ units/week $(\uparrow)$

$q = 100$ units

**Goal:** $\frac{dP}{dt} = ???$

Find eq$^n$ that relates $P$ and $q$ (this time, the problem gives it to us):

$$P(q) = 5q - 0.01q^2 - 1000$$

Now, differentiate but w.r.t. $t$:

$$\frac{dP}{dt} = 5\frac{dq}{dt} - 0.02q\frac{dq}{dt}$$

So,

$$\frac{dP}{dt} = 5(10) - 0.02(100)(10)$$

$$= 50 - 20 = 30$$

Thus, under these conditions,

$$\frac{dP}{dt} = \$30/\text{week}$$ (increase in profit)

12. Two products are competing. The sales of product A are related to the sales of product B according to the following formula: $3\sqrt{A} + 5\sqrt{B} = 55$. When 64 units of product B are being sold, the sales are increasing at a rate of 4 units/day. At what rate are the sales of product A changing?

A = sales of product A

B = sales of product B.

**Known:** B = 64 units

$\frac{dB}{dt} = 4$ units/day

**Goal:** $\frac{dA}{dt} = ??$

Relevant eqⁿ: $3\sqrt{A} + 5\sqrt{B} = 55$

$$3A^{1/2} + 5B^{1/2} = 55$$

$$\Rightarrow \frac{3}{2}A^{-1/2} \cdot \frac{dA}{dt} + \frac{5}{2}B^{-1/2}\left(\frac{dA}{dt}\right)$$

What A value do we use?

From original: $3\sqrt{A} + 5\sqrt{64} = 55$

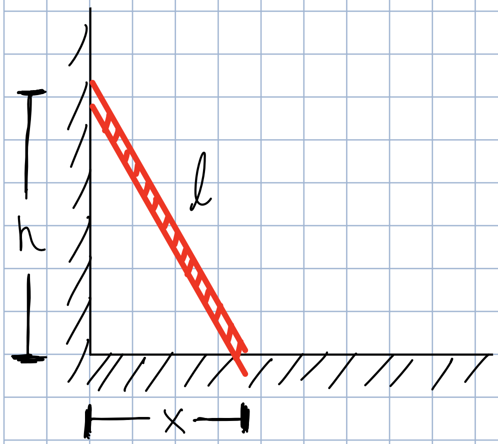
$$\Rightarrow 3\sqrt{A} + 40 = 55$$

$$A = 25$$

So, $$\frac{3}{2}A^{-1/2} \cdot \frac{dA}{dt} + \frac{5}{2}B^{-1/2}\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \frac{3}{2}\frac{1}{5}\frac{dA}{dt} + \frac{5}{2}\frac{1}{8}(4) \Rightarrow \frac{dA}{dt} = -4.17 \text{ units/day}$$

5. The base of a 50-foot ladder is being pulled away from a wall at a rate of 10 feet per second. How fast is the top of the ladder sliding down the wall at the instant when the base of the ladder is 30 feet from the wall?




$\ell$ = length of ladder

$h$ = height of top of ladder

$x$ = distance from base of ladder $\longrightarrow$ wall

**Known:** $\ell = 50$ ft, $x = 30$ ft

$$\frac{dx}{dt} = 10 \text{ ft/sec}$$

**Goal:** $\dfrac{dh}{dt} = ??$

Relevant eq$^n$: $x^2 + h^2 = \ell^2$

$$\Rightarrow \quad 2x \overset{\checkmark}{} \frac{dx}{dt}\overset{\checkmark}{} + 2h^{?} \frac{dh}{dt}^{?} = 2\ell^{\checkmark} \frac{d\ell}{dt}^{?}$$

For $h$: $x^2 + h^2 = \ell^2$

$$30^2 + h^2 = 50^2 \Rightarrow h = 40 \text{ ft}$$

For $\dfrac{d\ell}{dt}$: observe that $\ell$ doesn't change!

So, $\dfrac{d\ell}{dt} = 0$

So,

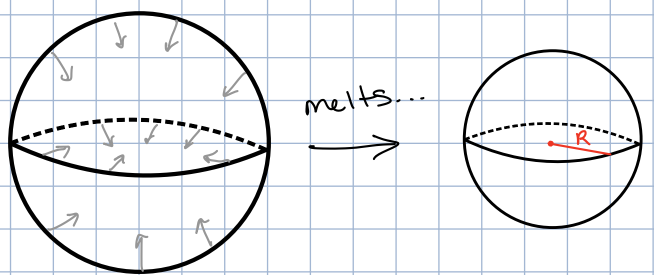
$$2x\left(\frac{dx}{dt}\right) + 2h\left(\frac{dh}{dt}\right) = 2l\left(\frac{dl}{dt}\right)$$

$$\Rightarrow 2(30)(10) + 2(40)\frac{dh}{dt} = 2(50)(0)$$

$$\Rightarrow 600 + 80\frac{dh}{dt} = 0$$

$$\Rightarrow \frac{dh}{dt} = -7.5 \text{ ft/sec}$$

11. A spherical snowball is melting so that its volume is decreasing at 1 cubic centimer per minute. At what rate is the diameter decreasing when the diameter is 10 centimeters? Note: If the radius of a sphere is $r$ then the diameter is $2r$ and the volume is $\frac{4}{3}\pi r^3$.




$V =$ volume

$D = 2R =$ diameter

**Known:** $\frac{dV}{dt} = -1 \text{ cm}^3/\text{min}$

$D = 10$ cm i.e. $R = 5$cm

**Goal:** $\frac{dD}{dt} = ??$

Relevant eq$^n$: $V = \frac{4}{3}\pi R^3$

$$\Rightarrow \frac{dV}{dt} = 4\pi R^2\left(\frac{dR}{dt}\right)$$

$$\Rightarrow \quad -1 = 4\pi(5)^2 \frac{dR}{dt}$$

$$\Rightarrow \quad \frac{dR}{dt} = \frac{-1}{100\pi} \quad cm/min$$

So, since $D = 2R$

$$\Rightarrow \frac{dD}{dt} = 2\frac{dR}{dt} = \frac{-1}{50\pi}$$

D is decreasing at a rate of $\frac{1}{50\pi}$ cm/min

**Example 14.3.** A street light is at the top of a 4-meter-tall pole. A dog, 0.8 meters in height, is running away from the pole along a straight path. If the dog is running at a speed of 6.4 m/sec., how fast is the dog's shadow lengthening when the dog is 20 meters from the pole?