- Exam this upcoming Monday

- Review session via Zoom on Sunday
  (6:30pm – 7:30 pm)

Section 10: Derivative Rules

→ quick derivative tools

List of Derivative Rules:

Unless otherwise specified, assume that f, g are arbitrary diff'ble functions.

- **Constant:**

Let $f(x)=c$, $c \in \mathbb{R}$. Then, $f'(x)=0$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{c-c}{h}=\lim_{h\to 0}\frac{0}{h}$$

$$=\lim_{h\to 0} 0 = 0$$

- **Identity:**

Let $f(x)=x$. Then, $f'(x)=1$.

• Constant Multiple:

Let $f(x)$ be diffble. Let $c \in \mathbb{R}$.

Then, $$\frac{d}{dx}\left[c \cdot f(x)\right] = c\,\frac{d}{dx}\left[f(x)\right]$$

• Sum/Difference:

Let f, g be diffble. Then,

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

• Product Rule: *

Let f, g be diffble. Then,

$$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = \frac{d}{dx}\left[f\right] \cdot g(x) + f(x)\,\frac{d}{dx}\left[g(x)\right]$$

• Quotient Rule: *

Let f, g be diffble. Then,

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}\left[f\right] \cdot g - f(x) \cdot \frac{d}{dx}\left[g(x)\right]}{g^2(x)}$$

Ir

$$\frac{d}{dx}\left[\frac{hi}{lo}\right] = \frac{lo \cdot d\,hi - hi \cdot d\,lo}{lo \cdot lo}$$

Y

• Power Rule:

Let $f(x) = x^n$. Then, $n \in \mathbb{R}$.

$$f'(x) = n x^{n-1}$$

Ex: Find the derivative of:

(a) $y = x^7 + 2x^2 - 1$

(b) $y = x^{\pi} - 2x$

(c) $y = (x^2+1)(x^{70} + 2x^9 + 8)$

(d) $y = \frac{4x+1}{x^2-2}$

(a) $y = x^7 + 2x^2 - 1$

$$y' = \frac{d}{dx}[x^7] + 2\frac{d}{dx}[x^2] - \frac{d}{dx}[1]$$

$$= 7x^6 + 2 \cdot 2x^1 \quad \text{---} \bigcirc$$

$$= 7x^6 + 4x$$

much faster than limit def^n!

(b) $y = x^{\pi} - 2x$

$$y' = \pi \cdot x^{\pi-1} - 2 \cdot 1$$

$$= \pi x^{\pi-1} - 2$$

"f" "g"

(c) $y = (x^2+1)(x^{70} + 2x^9 + 8)$

$$y' = f'(x) \cdot g(x) + f(x) g'(x)$$

$f'(x) = 2x$, $g'(x) = 70x^{69} + 18x^8$

$$= 2x(x^{70} + 2x^9 + 8) + (x^2+1)(70x^{69} + 18x^8)$$

(d) $y = \dfrac{4x+1}{x^2-2}$ → f, → g

quotient rule!

$$y' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$f'(x) = 4$

$g'(x) = 2x$

$$y' = \frac{4(x^2-2) - (4x+1)\,2x}{(x^2-2)^2}$$

Ex.:

3. Given $C(x) = 6x^2$ and $R(x) = 8x$. Find the marginal cost, marginal revenue, and marginal profit functions, and then find all the values of x for which the marginal profit is zero.

"marginal" → always means rate of change w.r.t to change in demand i.e. x

Think derivative!

$C'(x) =$ marginal cost $= 12x$

$R'(x) =$ marginal revenue $= 8$

Profit = Revenue − Cost.

$= 8x - 6x^2$

$P'(x) =$ marginal profit $= 8 - 12x = 0 \rightarrow x = \frac{2}{3}$