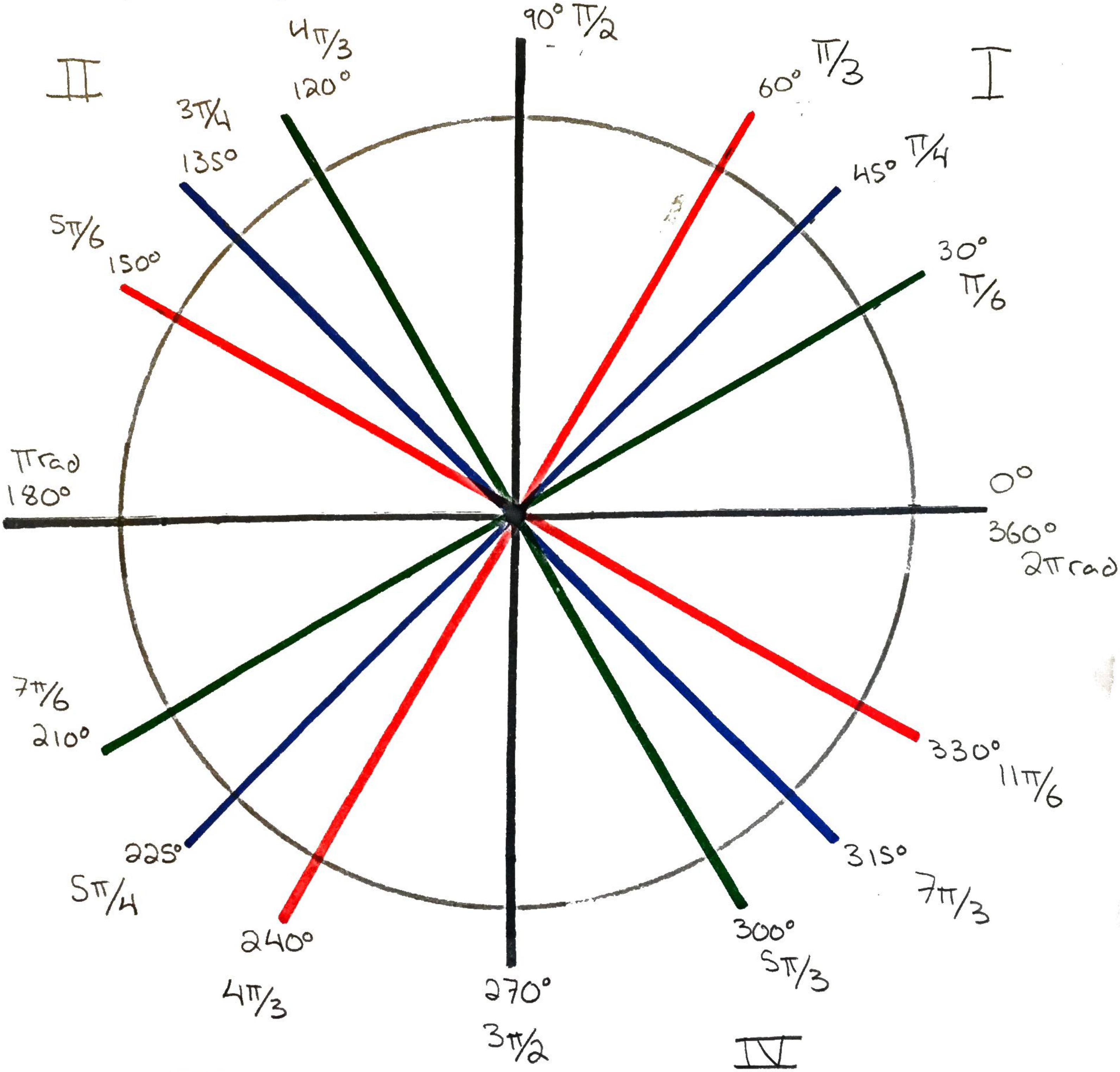
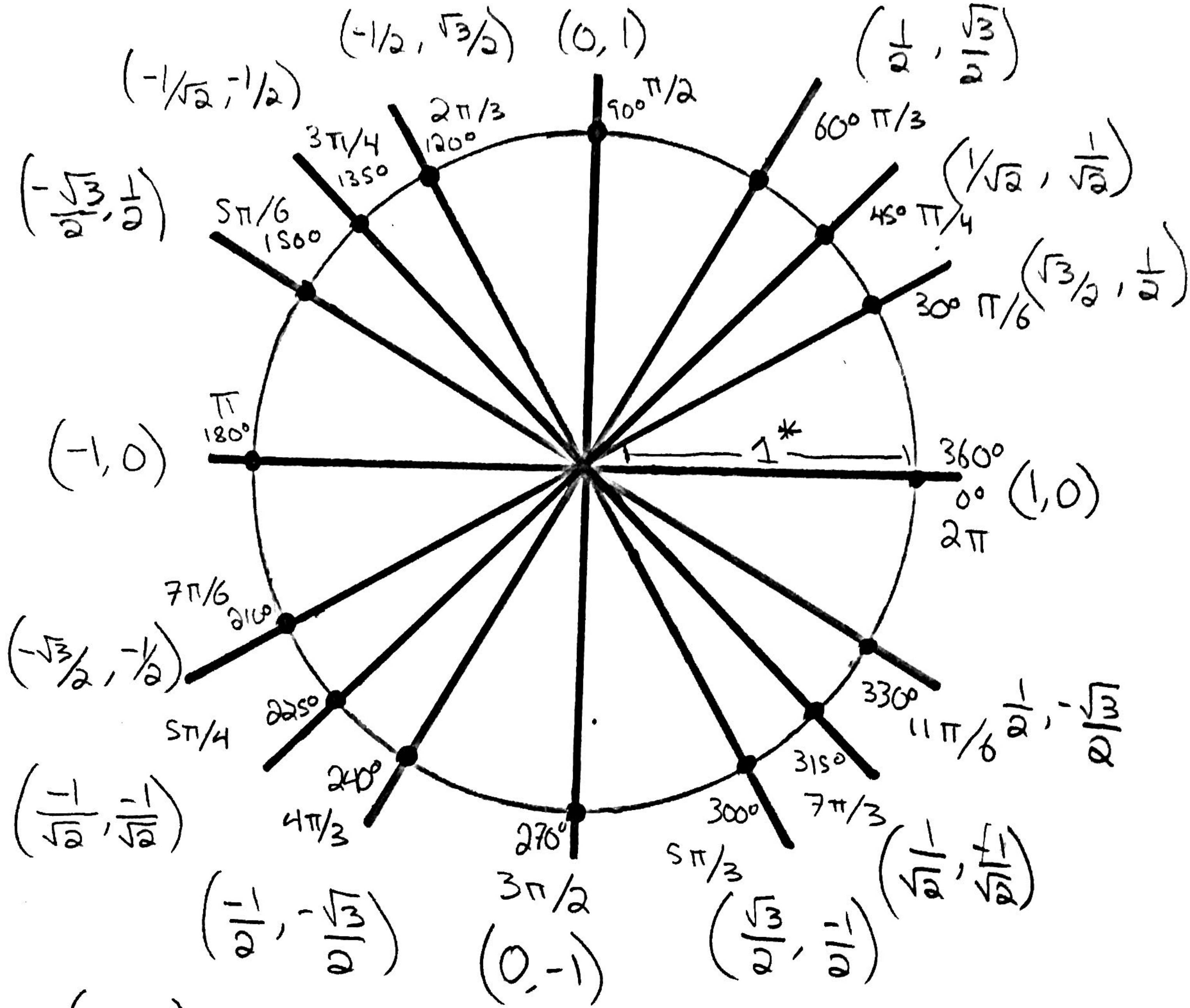
θ	0°	30°	45°	60°	90°	θ	0°	30°	45°	60°	90°
sinθ	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	cotθ	undef.	$\sqrt{3}$	1	$1/\sqrt{3}$	0
cosθ	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	secθ	1	$2/\sqrt{3}$	$\sqrt{2}$	2	undef
tanθ	0	$\sqrt{3}/3$	1	$\sqrt{3}$	undef.	cscθ	undef	2	$\sqrt{2}$	$2/\sqrt{3}$	1



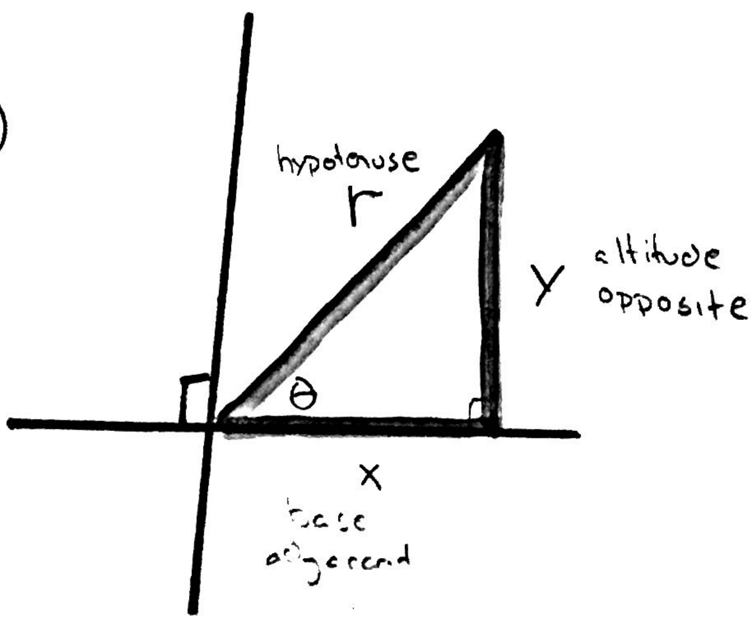
Dustin Gibian

*Radius 1 Unit Circle



(x, y)

(cosθ, sinθ)



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

Dustin Gibian

Given: $\tan\theta = -3/8$, $\sin\theta > 0$ 4/20/2018

S	A
T	C

lies in Quadrant 2

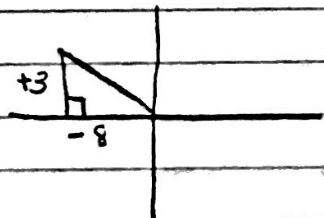
The trig ratios generate the values of the trig functions

$y = \text{trig}\,x$
- input angle
- output ratio

$y = \text{trig}^{-1}x$ (inverse)
- input ratio
- output angle

★ $y = f(x)$ which means trig(x) and $x = \theta$

Find the values of the other 5 trig ratios for this θ



$$\begin{array}{r} 64 \\ +\ 9 \\ \hline 73 \end{array}$$

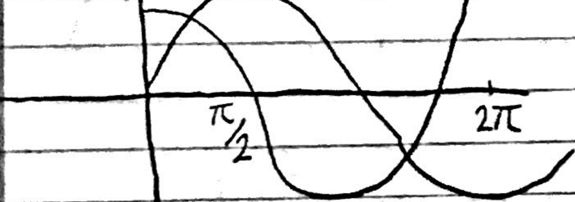
$a^2 + b^2 = c^2$
$(+3)^2 + (-8)^2 = c^2$
$9 + 64 = c^2$
$\sqrt{73} = \sqrt{c^2}$

$\sin\theta = 3/\sqrt{73}$, $\cos\theta = -8/\sqrt{73}$, $\csc\theta = \sqrt{73}/3$, $\sec\theta = \sqrt{73}/-8$,
$\cot\theta = 8/-3$

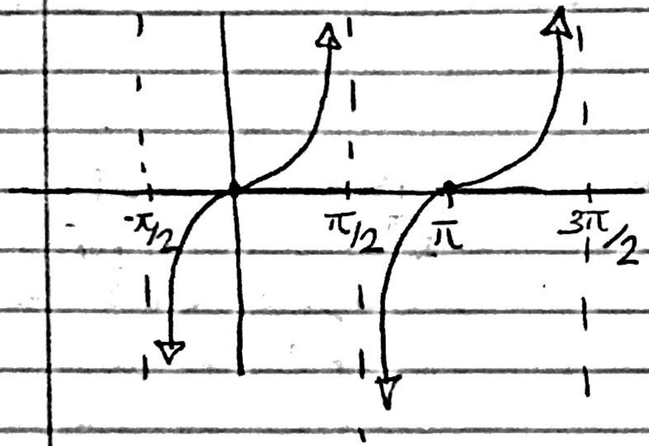
Reciprocal ID's

$\frac{1}{\sin\theta} = \csc\theta$ $\frac{1}{\cos\theta} = \sec\theta$ $\frac{1}{\tan\theta} = \cot\theta$

Know Features of sin, cos, tan



period: 2π
amp: 1
Dom: $\theta \in \mathbb{R}$
Range: $[-1, 1]$



per: π

amp: ℝ

Dom: $\theta \neq \frac{n\pi}{2}$, where n is odd

Range: ℝ

sin x, cos x

C: C/B - horizontal shift

D: vertical shift → Draw new midline before shifting it

A(Bx+C)+D

|A|: amp

B: per = 2π/B

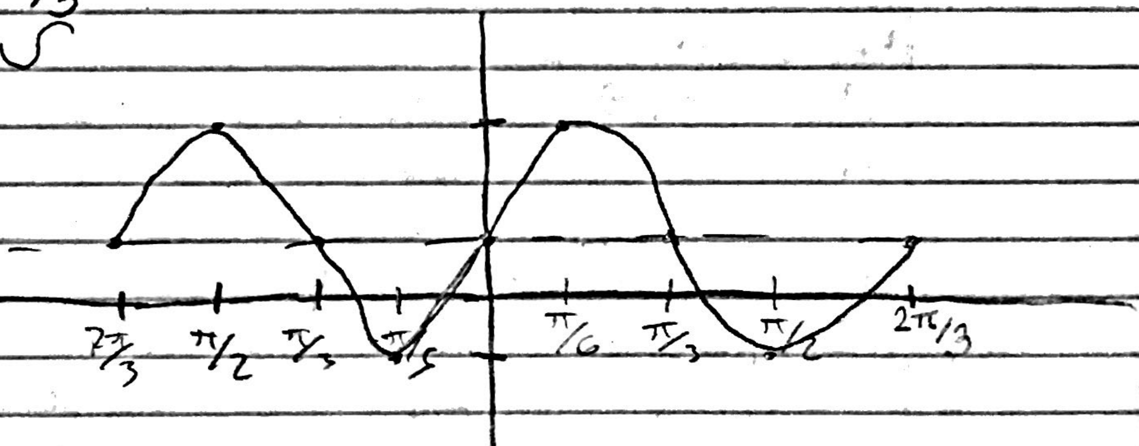
$y = -2\sin(3x) + 1$

amp: 2

range: $[-2, 2] \xrightarrow{+1} [-1, 3]$

period: 2π/3

shape: ∽



Order of Transformations

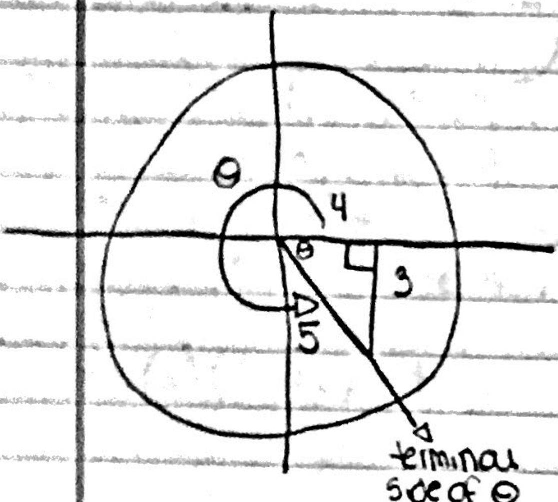
$A(Bx+C)+D = \sin x$

1. Draw mother function
2. Deal w/ period (adjust $2\pi/B$)
3. Horizontal shift (C/B)
4. Reflection if needed
5. Vertical Shift

Going Over Quiz Question

4/23/2018

$\sin\theta = -3/5$, $\tan\theta < 0$ (both (-) so it has to be Quad 4)



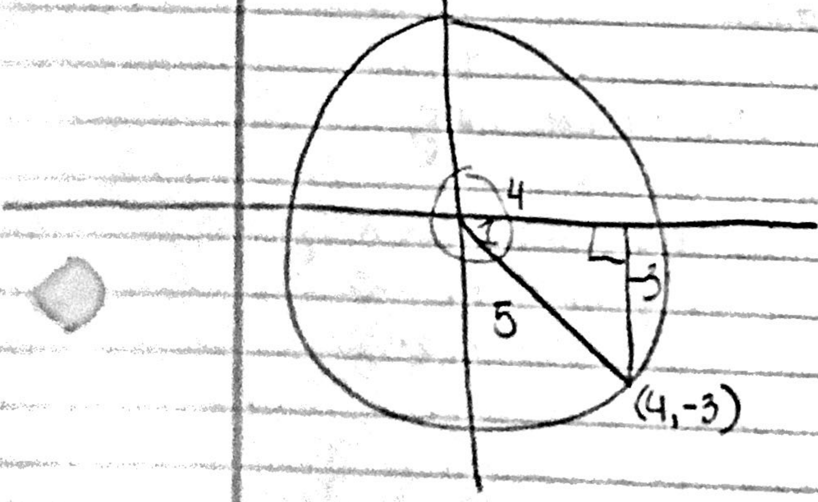
To find the missing side...

$\sqrt{25-9} = \sqrt{16} = 4$

Famous Pythag. Triples

$a^2+b^2=c^2$

- □ $3^2+4^2=5^2$
- □ $5^2+12^2=13^2$



$$\left(\frac{x}{r}, \frac{y}{r}\right)$$

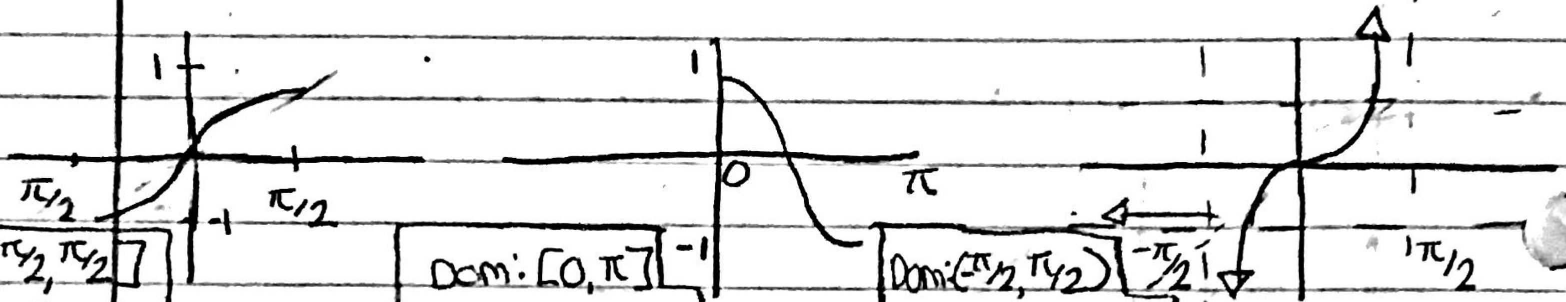
$$\downarrow$$

$$\left(\frac{4}{5}, \frac{-3}{5}\right)$$

coordinates where terminal side of θ intersects unit circle

Inverse Trig

$\sin\theta$ $\cos\theta$ $\tan\theta$



Dom: $[-\pi/2, \pi/2]$
Range: $[-1, 1]$

Dom: $[0, \pi]$
Range: $[-1, 1]$

Dom: $(-\pi/2, \pi/2)$
Range: $(-\infty, \infty)$

When we restrict the domain of the $\sin\theta$, $\cos\theta$, $\tan\theta$ fcns, we can define their inverses

Steps $\qquad\qquad$ ★ $(f \cdot f^{-1})(x) = x$

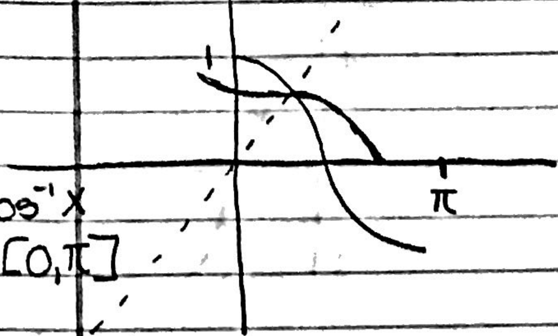
① $y = \sin x \longrightarrow$ Given
② $x = \sin y \longrightarrow$ Flip x & y
③ $\sin^{-1} x = \sin^{-1}(\sin y) \longrightarrow$ take the inverse of both sides
④ $\sin^{-1} x = y \longrightarrow$ final answer

$\sin^{-1} x$ or $\arcsin x$ mean the same thing

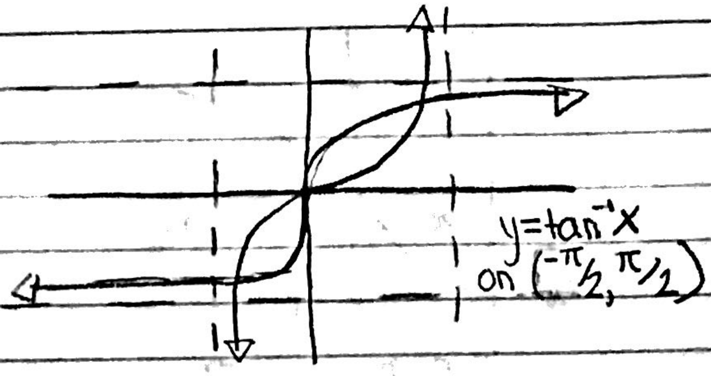
$y = \sin x$; $y = \cos x$; $y = \tan x$
- input: angle
- output: ratio

$y = \sin^{-1} x$; $y = \cos^{-1} x$; $y = \tan^{-1} x$
- input: ratio
- output: angle

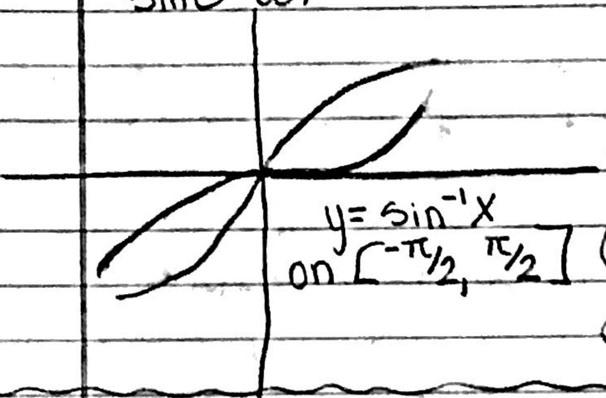
$\cos\theta$ w/ (-1)



$\tan\theta$ w/ (-1)



$\sin\theta$ w/ (-1)



θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	undefined

$\sin \pi/4 = 1/\sqrt{2}$ input angle
$\sin^{-1} 1/\sqrt{2} = \pi/4$ input ratio

$\cos \pi/6 = \sqrt{3}/2$ input angle
$\cos^{-1}(\sqrt{3}/2) = \pi/6$ input ratio

Example Problem

find $\sin^{-1}(1/\sqrt{2}) = \pi/4$ or $\cancel{3\pi/4}$ → This answer doesn't work b/c it is out of the domain

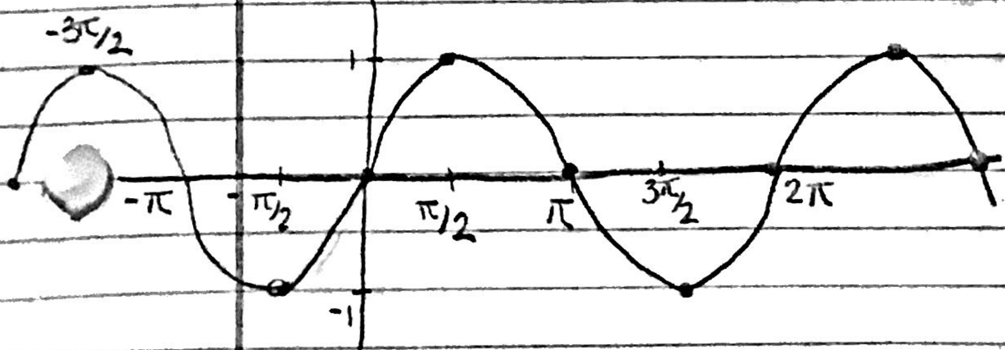
find $\sin^{-1}(-\sqrt{3}/2) = -\pi/3$

Hollistic Approach to Trig Equations 4/24/2018

$3x-2=7$
$x=3$

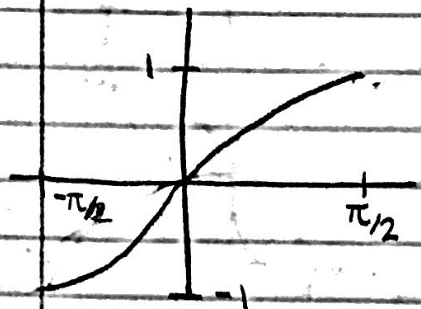
$\sin x = 1$

$x = -3\pi/2,\ \pi/2$; $x = \pi/2 \pm 2\pi n$, where n is an integer

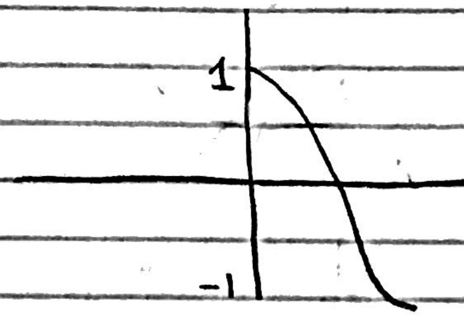


★ Sec 8.6 (inverse trig fcns), 8.9 (solving trig eqns)

$y = \sin x$



$y = \cos x$



$[-\pi/2, \pi/2]$ $\qquad$ $[0, \pi]$

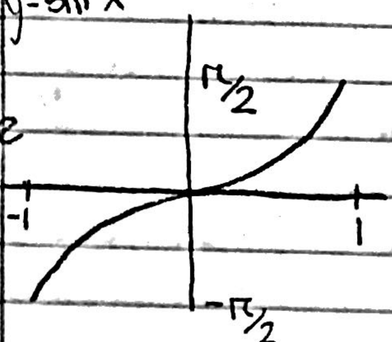
$y = \sin^{-1} x$ exists on $[-\pi/2, \pi/2]$ of $y = \sin x$

$y = \cos^{-1} x$ exists on $[0, \pi]$ of $y = \cos x$

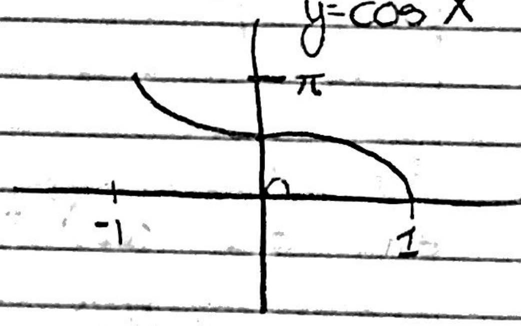
Should draw inverses on separate graph

*The domain of the standard becomes the range of the inverse

$y = \sin^{-1} x$



$y = \cos^{-1} x$



x
$-\pi/2$
$-\pi/3$
$-\pi/4$
$-\pi/6$
0
$\pi/6$
$\pi/4$
$\pi/3$
$\pi/2$

Table on Next Page ...

x	$\sin x$	$\cos x$	$\tan x$
$-\pi/2$	-1	XX	$-\infty$
$-\pi/3$	$-\sqrt{3}/2$	XX	$-\sqrt{3}$
$-\pi/4$	$-1/\sqrt{2}$	XX	-1
$-\pi/6$	$-1/2$	XX	$-1/\sqrt{3}$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	∞
$4\pi/3$	XX	$-1/2$	XX
$3\pi/4$	XX	$-1/\sqrt{2}$	XX
$5\pi/6$	XX	$-\sqrt{3}/2$	XX
π	XX	-1	XX

* make sure you can correspond every $\sin^{-1}$(ratio) w/ its angle

Examples

1. $\sin^{-1}(1/2) = \pi/6$
2. $\sin^{-1}(-1) = -\pi/2$
3. $\sin^{-1}(\sqrt{3}/2) = \pi/3$
4. $\cos^{-1}(0) = \pi/2$
5. $\cos^{-1}(1) = 0$
6. $\cos^{-1}(-1/2) = 4\pi/3$
7. $\tan^{-1}(\sqrt{3}) = \pi/3$
8. $\tan^{-1}(-1) = -\pi/4$
9. $\tan^{-1}(0) = 0$

* $f(f^{-1}(x)) = x$
$f^{-1}(f(x)) = x$
$f \circ f^{-1} = f^{-1} \circ f$

Trig Identities

1. $\frac{\sin\theta}{\cos\theta} = \tan\theta$
2. $\sin^2\theta + \cos^2\theta = 1$
"Pythag identity"

To be continued ...

ratio　　angle　　ratio

- $\sin(\sin^{-1}(1/2)) = 1/2 = \sin(\pi/6) = 1/2$
- $\cos(\cos^{-1} x) = x$

$\cos(\underbrace{\cos^{-1}(\underbrace{x}_{\text{ratio}})}_{\text{angle}})$

ratio

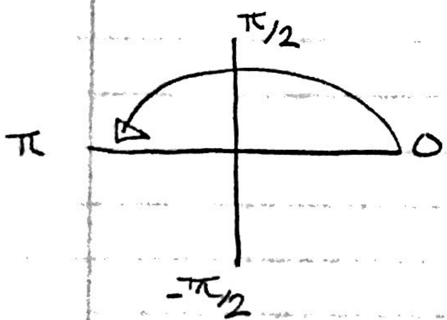
$\cos(\cos^{-1}(1/2)) = \cos(\pi/3) = 1/2$

$\tan(\tan^{-1}(\infty)) = \tan(\pi/2) = \infty$

$\tan(\tan^{-1}(-\sqrt{3})) = \tan(-\pi/3) = -\sqrt{3}$

- The easy compositions are $\text{trig}(\text{trig}^{-1}(x)) = x$, you'll always get x back, the ratio
- The harder problem is: $\text{trig}^{-1}(\text{trig}\,x) = x$
  only if $x \in$ restricted domain of the trig function

* $\sin^{-1}(\sin \pi) \neq \pi$



$\sin^{-1}(0) = \boxed{0}$

$$-\pi/2 \le \sin^{-1}(x) \le \pi/2$$
$$0 \le \cos^{-1}(x) \le \pi$$
$$-\pi/2 < \tan^{-1}(x) < \pi/2$$

$$-1 \le \sin(x) \le 1$$
$$-1 \le \cos(x) \le 1$$
$$-\infty < \tan(x) < \infty$$

$\cos^{-1}(\cos \pi/6) = \pi/6$

$\cos^{-1}((\cos \pi/4)) = \pi/4$

$\cos^{-1}(\cos(-\pi/3))$

$\cos^{-1}(+1/2) = \pi/3$

- $\sin(\cos^{-1}(1))$
  $\sin(0) = 0$
- $\tan^{-1}(\sin(\pi/3))$
  $\tan^{-1}(\sqrt{3}/2) = ?$
- $\cos(\sin^{-1}(-\pi/4))$
  $\cos(-1/\sqrt{2}) = 3\pi/4$

Fundamental Trig Identities

- ☐ $\tan x = \frac{\sin x}{\cos x}$
- ☐ $\cot x = \frac{1}{\tan x}$
- ☐ $\sec x = \frac{1}{\cos x}$
- ☐ $\csc x = \frac{1}{\sin x}$

"cofcns trig IDs"

- ☐ $\sin x = \cos(\pi/2 - x)$
- ☐ $\sec x = \csc(\pi/2 - x)$
- ☐ $\tan x = \cot(\pi - x)$

Pyth ID's

- ☐ $\sin^2 x + \cos^2 x = 1$
- ☐ $1 + \cot^2 x = \csc^2 x$ (when you ÷ everything by $\sin^2 x$)
- ☐ $\tan^2 x + 1 = \sec^2 x$ (when you divide everything by $\cos^2 x$)