3/9/2018

Ch. 7 - Solving inequalities of quadratic, rational, + absolute value fcns

- When you solve an equation, you get a finite # of answers (x= _).
- When solve an inequality you get an infinite solution set (interval notation) - (a,b), $[a,\infty)$, $(-\infty,\infty)$ ...

Properties of Inequalities

① If $a \le b$ then $a \pm c \le b \pm c$

② If $a < b$ then $-a > -b$
(mult. or dividing by a (-) flips the sign)

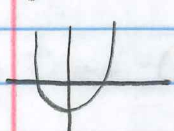
③ If $a > b$ then $\frac{1}{a} < \frac{1}{b}$

* ④ If $ab = 0$ then a or b is 0
If $ab > 0$ then both $a,b > 0$ (+) or both $a,b < 0$ (-)

⑤ If $a/b > 0$ then $a,b > 0$ or $a,b < 0$

Use ④ as foundation for quadratic + rational inequalities
(parabolas U) (asymptotes)

$y = x^2 - 4$ values $= 0$ (2 & -2)



$y > 0$ for which x → $(-\infty,-2) \cup (2,\infty)$

Algebraic Solution

$x^2 - 4 > 0$ $\xrightarrow{\text{DO NOT}}$ $x-2>0$, $x+2>0$

Factor DO: $(x-2)(x+2) > 0$ Roots: $x = -2, 2$

$x-2 > 0$ & $x+2 > 0$

or

$x-2 < 0$ & $x+2 < 0$

(-3
$\checkmark (-3-2)(-3+2) > 0$
$(-5)(-1) > 5$

$(3-2)(3+2) > 0$
$(1)(5) > 0 \checkmark$

$$\boxed{\text{Solution} \quad (-\infty,-2) \cup (2,\infty)}$$

these #s between do not work

To solve a quadratic inequality find the roots of the equation, put them on the no. line, check intervals into original equation

Rational inequalities

$\frac{x}{x+1} > 0$ DO NOT CROSS MULT.

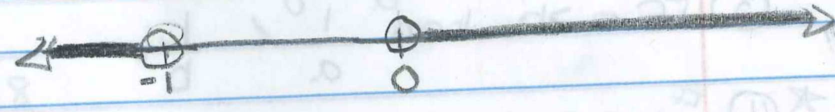
$\frac{a}{b} > 0$ then $a, b > 0$ or $a, b < 0$

To solve a rational inequality, find the roots of the top and bottom, plot them on the # line, and test the intervals

$x = 0$ $\quad$ $x + 1 = 0$

$\quad\quad\quad -1 \quad -1$

$\quad\quad\quad x = -1$



$>, <$ or known as strict inequalities

$\frac{-2}{-2+1} > 0$ $\quad 2 > 0$ ✓ | x $\frac{-}{+}$ = - | $\frac{1}{1+1} > 0$ ✓

$\boxed{(-\infty, -1) \cup (0, \infty)}$

$y = 2x - 5 < 0$

$2x - 5 = 0$

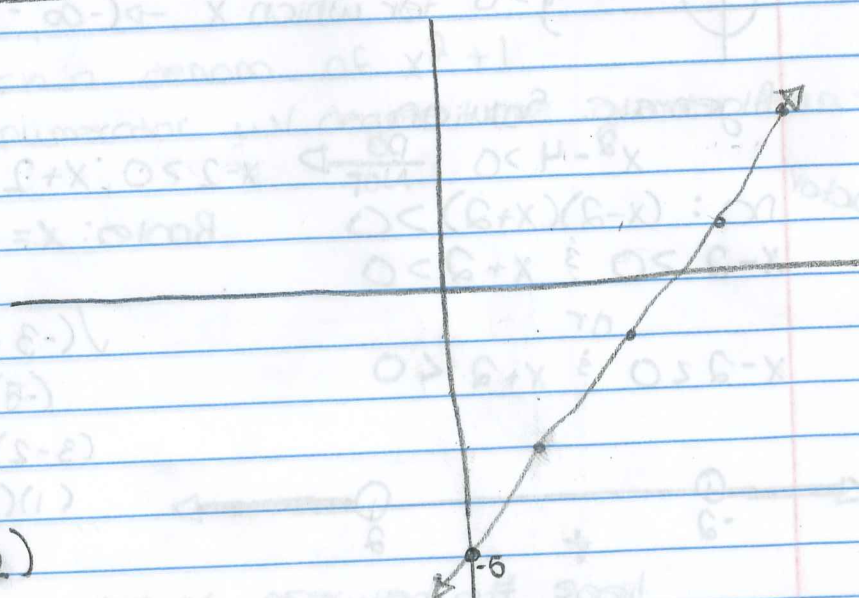
$\quad +5 \quad +5$

$\frac{2x}{2} = \frac{5}{2}$

$x = \frac{5}{2}$

Algebraic : $x < 5/2$

Interval : $(-\infty, 5/2)$



⑥ Take ④, ⑤, and consider less than

If $ab < 0$ then $a < 0$ and $b > 0$ OR $a > 0$ and $b < 0$

Also if $\frac{a}{b} < 0$ then $a < 0$, $b > 0$ OR $a > 0$, $b < 0$

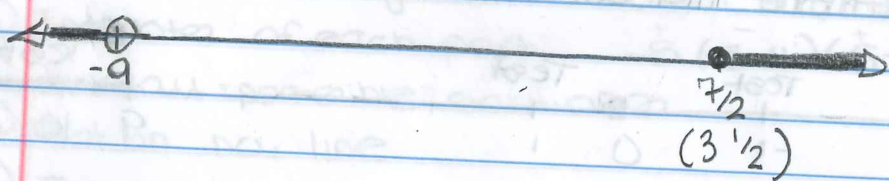
Ex) $\frac{7-2x}{x+9} \leq 0$

① Roots

Top: $7 - 2x = 0$
$-7 \qquad -7$
$\frac{-2x}{-2} = \frac{-7}{-2}$
$\boxed{x = 7/2}$

Bottom: $x + 9 = 0$
$-9 \quad -9$
$\boxed{x = -9}$

② No. Line



$7 - 2(7/2) = 0$ ✓

$-9 + 9 = 0$ X do not want Ø on bottom

③ Test (-10, 0, 4)
I1 I2 I3

$\frac{7-0}{0-9} = \frac{7}{9} > 0$

So not in solution set

$\frac{7-2(-10)}{-10+9} = \frac{27}{-1} < 0$ ✓

$\frac{7-2(4)}{4+9} \leq 0$ ✓

$\boxed{(-\infty, -9) \cup [7/2, \infty)}$

★ You need the roots even if they are not in the solution

Pre-Step to ① Roots, ② No. line, ③ Test

Make sure your inequality is <u>set against</u> zero before beginning the other steps

<u>STEPS</u>

0. Set against $\varnothing$
1. Roots
2. No. line
3. Test (into factored problem)

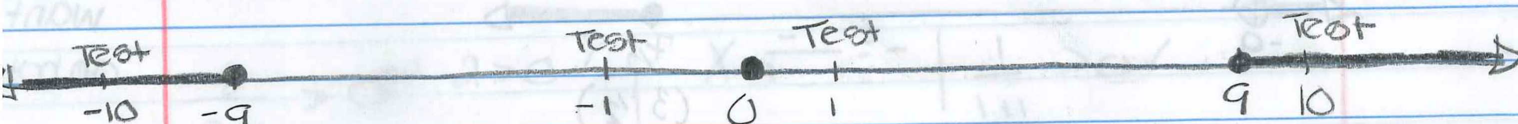
★ Ex) $x^4 \geq 81x^2$ KNOW THIS EXAMPLE

$x^4 - 81x^2 \geq 0$ $\qquad$ $x^2 = 0 ; x = 0$

$x^2(x^2 - 81) \geq 0$ $\qquad$ $x + 9 = 0 ; x = -9$

$x^2(x+9)(x-9) \geq 0$ $\qquad$ $x - 9 = 0 ; x = +9$

u will always have 1 more interval than you have roots



$-10)^2(-10-9)(-10+9) \geq 0$

$+ \quad - \quad -$

$+ \geq 0$ ✓

$(10)^2(10-9)(10+9) \geq 0$

$+ \quad + \quad +$

$+ \geq 0$ ✓

$$\boxed{(-\infty, -9] \cup \{0\} \cup [9, \infty)}$$

$-1)^2(-1-9)(-1+9) \geq 0$

$+ \quad - \quad +$

$- \geq 0$ X

$1)^2(1-9)(1+9) \geq 0$

$+ \quad - \quad +$

$- \geq 0$ X

$\frac{x+3}{x-4} \le 2$ $x^3 - 3x^2 - 4x \ge 0$ 3/12/2018

① Set against zero

$\frac{x+3}{x-4} - \frac{2}{1}^{(x-4)} \le 2$ $x^3 - 3x^2 - 4x \ge 0$

② Combine as needed

$\frac{x+3-2x+8}{x-4} \le 0$ $x(x^2-3x-4) \ge 0$

$x(x-4)(x+1) \ge 0$

$x=0$ $x=4$ $x=-1$

$\frac{-x+11}{x-4} \le 0$

Top: 11

Bottom: 4

$-2(-2-4)(-2+1) \ge 0$

$-\frac{1}{2}(-\frac{1}{2}-4)(-\frac{1}{2}+1) \ge 0$

$1(1-4)(1+1) \ge 0$

$5(5-4)(5+1) \ge 0$

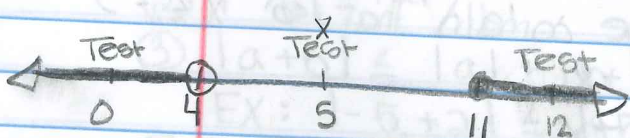
combine into 1 step

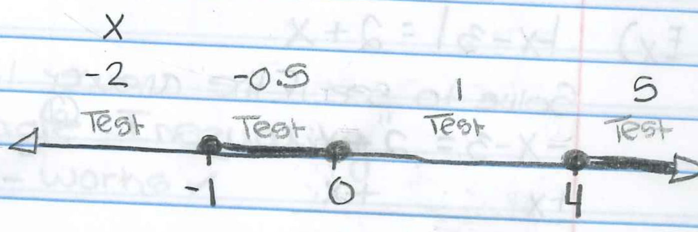
③ Find roots of each part

④ Find all possible solutions

⑤ Plot on no. line

⑥ Test pts. w/in zero





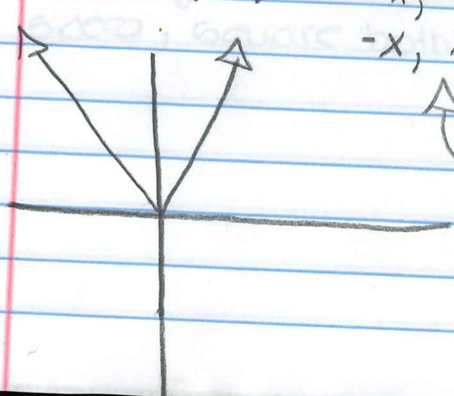
$(-\infty, 4) \cup [11, \infty)$ $[-1, 0] \cup [4, \infty)$

$f(x) = |x| \rightarrow x, \; x \ge 0$

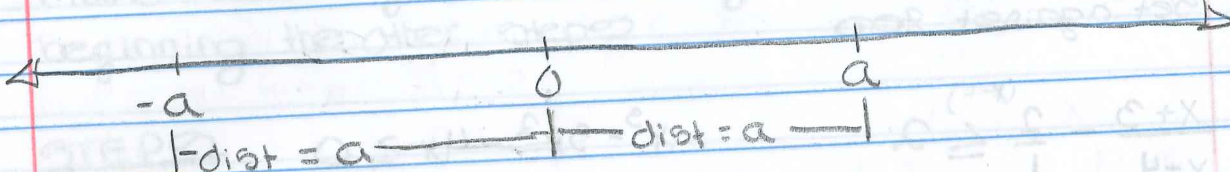
$-x, \; x < 0$

identical function: $g(x) = \sqrt{x^2}$

b/c it has the same domain and the same $\boxed{\text{rule}}$



Def: $|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$ where a is a number



Ex) $|7| = 7$; $|-11| = 11$

★Piecewise fcns: domains are broken up

Def: $|f(x)| = \begin{cases} f(x), & \text{where } f(x) \geq 0 \\ -f(x), & \text{where } f(x) < 0 \end{cases}$

Ex) $|x-3| = 9$

Break it up into 2 separate cases

Case 1 (remove | |)

$x - 3 = 9$
$+3 \quad +3$
$x = 12$

Case 2 (negate expression in | |)

$-x + 3 = 9$
$-3 \quad -3$
$-x = 6$
$\overline{-1} \quad \overline{-1}$
$x = -6$

Ex) $|-x-3| = 2 + x$

Solve to see if the answer lies in the domain that is $x \leq -3$

$-x - 3 = 2 + x$
$+x \quad +x$
$-3 = 2 + 2x$
$-2 \quad -2$
$-5 = 2x$
$\overline{2} \quad \overline{2}$
$-5/2 = x$
↓
$\notin$ Dom

② $\cancel{x} + 3 = 2 + \cancel{x}$
$-\cancel{x} \quad -x$

no solution

Always check your solutions back into the equation

Ex) $|x-5| = 3$

$x-5=3$ $\quad$ $-x+5=3$

$+5\ +5$ $\quad$ $-5\ -5$

$x=8$ $\quad$ $-x=-2$

$\frac{-x}{-1} = \frac{-2}{-1}$

$x=2$

After the check, both work

$|8-5|=3$ ✓

$|2-5|=3$ ✓

Ex) $|2x-1| = 4x-9$

$2x-1=4x-9$ $\quad$ $-2x=4x-9$

$-2x\ -2x$ $\quad$ $+2x\ +2x$

$-1=2x-9$ $\quad$ $1=6x-9$

$+9\ +9$ $\quad$ $+9\ +9$

$\frac{8}{2}=\frac{2x}{2}$ $\quad$ $\frac{10}{6}=\frac{6x}{6}$

$4=x$ $\quad$ $5/3=x$

$\frac{5}{3} \cdot \frac{2}{1} - \frac{10}{3} - \frac{1^{(3)}}{1(3)}$

$\frac{4}{1} \cdot \frac{5}{3}\ \frac{20}{3} - \frac{9^{(3)}}{1(3)}$

Check

$|2(4)-1| = 4(4)-9$

$7=7$ ✓

Check

$\frac{7}{3} \neq -\frac{7}{3}$

★ Just one solution for this equation

Properties of Absolute Value Equations

1. $|ab| = |a||b|$
2. $|a/b| = |a|/|b|$
3. $|a+b| \leq |a|+|b|$ "Triangle Inequality"
   EX: $|-5+2| \leq |-5|+|2|$ works ✓
4. $|a-b| \geq |a|-|-b|$
5. if $|a|=|b|$ then $|a|^2=|b|^2 \rightarrow a^2=b^2$

when you see a problem w/ absolute values on both sides, square both sides to remove $|\ |$ completely

Absolute Value - Equations

① $|f(x)| = g(x)$

then ... $f(x) = g(x)$ when $f(x) \geq 0$

and ... $f(x) = -g(x)$ when $f(x) < 0$

② $|f(x)| = |g(x)|$

square both sides to solve, removing | |.

(since $|a|^2 = a^2$)

$f(x)^2 = g(x)^2$ etc. (solve the quadratic or other)

③ $|f(x)| = |g(x)| + C$

$f(x) = g(x) + C$

$-f(x) = g(x) + C$

$f(x) = -g(x) + C$

$-f(x) = -g(x) + C$

with accompanying domain restrictions, but we find it's easiest (often times) to check answers found in each case rather than check against the domain

Hw2*) $|x^2 + x| = |x - 15|$

$x^2 + x = x - 15$

$x^2 + x = -x + 15$

$-x^2 - x = x - 15$

$-x^2 - x = -x + 15$

same

so, you only have two cases really

Case 1

$x^2 + \cancel{x} = \cancel{x} - 15$

$-x \quad -x$

$x^2 = -15$

$x^2 \neq -15$

R

Case 2

$x^2 + x = -x + 15$

$+x \quad -15 \quad +x \quad -15$

$x^2 + 2x - 15 = 0$

$(x-3)(x+5) = 0$

$x = 3$

$x = -5$

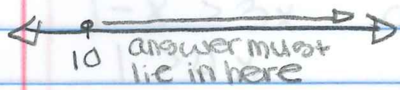
-10 F1

HW) Ex) $|x-10| = x^2 - 10x$

① $x - 10 = x^2 - 10x$

when $x - 10 \geq 0$

$x \geq 10$



$x - 10 = x^2 - 10x$

$\underline{-x + 10 \qquad -x + 10}$

$x^2 - 11x + 10 = 0$

$(x-10)(x-1) = 0$

$x = 10$, $x = 1$ not in domain

Reject

② $-x + 10 = x^2 - 10x$

$+x \quad -10 \qquad +x \quad -10$

$x^2 - 9x - 10 = 0$

$(x-10)(x+1) = 0$

$x = 10 \quad x = -1$ ✓

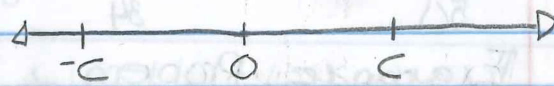
$\boxed{\text{Answers: } x = 10, -1}$

$|x| = c$

c units away from zero

x lies exactly c units from zero

$|x| < c$



x lies w/in c units of zero

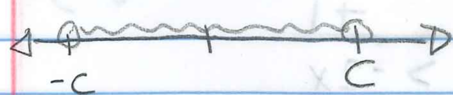
$|x| > c$

x lies outside c units of zero

★ Must understand the meaning behind the algorithms

Ex) $|x| < c$ $\qquad$ $|f(x)| < c$

graph



alg. $-c < x < c$

interval $(-c, c)$

must show all 3 versions for everything

Ex) $|x| \geq c$ and $|f(x)| > c$

$-c$ c

graph

alg. $x \leq -c$ or $x \geq c$

interval $(-\infty, -c] \cup [c, \infty)$

Ex) $\left|\dfrac{x+8}{3}\right| < 14$ or $\dfrac{x+8}{3} < 14$ and $\dfrac{x+8}{3} > -14$

$x < 34$ $x > -50$

$-14 < \dfrac{x+8}{3} < 14$ (3) (3)

-50 34

$-42 < x+8 < 42$

$-8 \quad -8 \quad -8$

$-50 < x < 34$

$\boxed{\begin{matrix}(-50, 34) \\ -50 < x < 34\end{matrix}}$

50 34

Example Problem 7.3

$\left|\frac{1}{x} - 1\right| < 3$

$-3 < \frac{1}{x} - 1 < 3$

$\frac{1}{x} - \frac{1}{1} < 3$ and $\frac{1}{x} - \frac{1}{1} > -3$

Clear the denom

this is all for $x > 0$

$1 - x < 3x$ $\quad$ $1 - x > -3x$

$+x \quad +x$ $\quad$ $+1 \quad +1$

$\frac{1}{4} < \frac{4x}{4}$ $\quad$ $\frac{1}{-2} > \frac{-2x}{-2}$

$x > \frac{1}{4}$ $\quad$ $-\frac{1}{2} < x$

$-1/2$ $1/4$ $(1/4, \infty)$

~~$(-1/2, 1/4)$~~

$\frac{1}{x} - 1 < 3$ and $\frac{1}{x} - 1 > -3$

where $x < 0$

$1 - x > 3x$ and $1 - x < -3x$

$1 > 4x$ $\qquad$ $1 < -2x$

$\frac{1}{4} > x$ and $-\frac{1}{2} > x$

Final answer in Interval Notation

$(-\infty, -\frac{1}{2}) \cap (\frac{1}{4}, \infty)$

.3 HW 9) $\left|\frac{x+3}{x+1}\right| \leq 3$

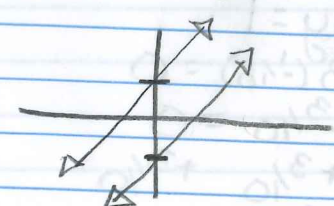
when you cross mult. you must consider both

$x - 1 < 0$ and $x - 1 > 0$

<u>Graphing Equations + Inequalities + Systems of Both</u>

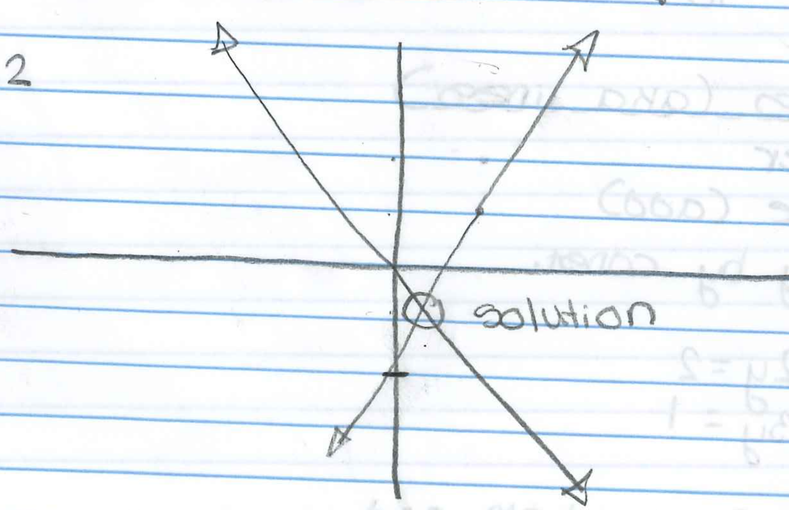
"Solve" system (more then 1 equation) means "find the point of intersection"

$$\begin{cases} x+1 = y \\ x-1 = y \end{cases}$$



parallel lines
no solution

$$\begin{cases} y = 3x-2 \\ y = -x \end{cases}$$



all lines represent vectors → both magnitude (size) and direction

- you can reorder them
- combine them
- mult. them by a constant

scale (scalar is a #)

Solve System of Equations w/ Algebra:

Choose 1 of 2 Techniques

1. Substitution (of one equation into other)
2. Elimination (of one variable to find the other)

Ex) $\begin{cases} x+2y=6 \\ x-2y=4 \end{cases}$

brace - tells us to find sol. of system (pt. of interaction)

elimination

$$\begin{aligned} x+\cancel{2y} &= 6 \\ +\ x-\cancel{2y} &= 4 \\ \hline 2x &= 10 \\ \tfrac{2x}{2} &= \tfrac{10}{2} \\ x &= 5 \end{aligned}$$

$$\begin{aligned} x+2y &= 6 \\ 5+2y &= 6 \\ -5 \quad & \quad -5 \\ \hline 2y &= 1 \\ \tfrac{2y}{2} &= \tfrac{1}{2} \\ y &= \tfrac{1}{2} \end{aligned}$$

$\boxed{(5, \tfrac{1}{2})}$ The two lines intersect here

$-6+-4$

Ex) $\begin{cases} x+3y=0 \\ 2x-4y=1 \end{cases} \xrightarrow{\text{mult. by } -2} -2(x+3y=0) =$

$$\begin{aligned} -\cancel{2x} - 6y &= 0 \\ +\ \cancel{2x} - 4y &= 1 \\ \tfrac{-10y}{-10} &= \tfrac{1}{-10} \\ y &= -\tfrac{1}{10} \end{aligned}$$

(b/c it's $0 \cdot 2$)

$$\begin{aligned} x+3(-\tfrac{1}{10}) &= 0 \\ x + -\tfrac{3}{10} &= 0 \\ +\tfrac{3}{10} \quad & \quad +\tfrac{3}{10} \\ x &= \tfrac{3}{10} \end{aligned}$$

$\boxed{(\tfrac{3}{10}, -\tfrac{1}{10})}$

Vectors (aka lines)

1. Reorder
2. Combine (add)
3. multiply by const.

$\begin{cases} 3x-2y=2 \\ 2x-3y=1 \end{cases}$

have to find the LCM and make them opposite signs