HW   Sec 33   Example HW problems

#1(a) $\int_{-1}^{1}(x^2+2)\,dx = \frac{x^3}{3}+2x\Big|_{-1}^{1} = \frac{1}{3}+2-\left(-\frac{1}{3}+2(-1)\right)$

$= \frac{1}{3}+2+\frac{1}{3}+2 = \boxed{14/3}$

d) $\int_{0}^{1} 6\left(4\sqrt{x}-3x\sqrt{x}\right)dx$

I showed intermediate steps of $\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

$= 6\int_{0}^{1}\left(4x^{1/2}-3x^{3/2}\right)dx = 6\int_{0}^{1}4x^{1/2}\,dx - 6\int_{0}^{1}3x^{3/2}\,dx$

$= 24\frac{x^{3/2}}{3/2}\Big|_{0}^{1} - \frac{18x^{5/2}}{5/2}\Big|_{0}^{1} = \frac{48}{3}x^{3}\Big|_{0}^{1} - \frac{36}{5}x^{5/2}\Big|_{0}^{1}$

$= 16(1^{3/2}) - 0 - \frac{36}{5}(1^{5/2}) - 0 = 16 - \frac{36}{5} = \boxed{\frac{44}{5}}$

c) $\int_{1}^{3}\left(1+\frac{1}{x}+\frac{1}{x^2}\right)dx = \left[x+\ln|x|+\frac{x^{-1}}{-1}\right]_{1}^{3}$

$= \left(3+\ln 3-\frac{1}{3}\right)-\left(\cancel{1}+\underbrace{\ln|1|}_{0}-\cancel{1}\right)$

$= \boxed{2\tfrac{2}{3}+\ln 3}$

h) $\int_{-1}^{0} 2x\sqrt{x+1}\,dx = 2\int_{-1}^{0}\underbrace{x\sqrt{x+1}}\,dx$

u-subsitition for both x + x+1 as follows:

$2\int_{\boxed{0}}^{\boxed{-1}}(u-1)\,u^{1/2}\,du \longleftarrow \boxed{u = x+1}$ $x = u-1$

$du = dx$

rewritten as u values: $x = -1,\ u = -1+1 = 0$

$x = 0,\ u = 0-1 = -1$

$$2\int_0^{+1}(u-1)(u)^{1/2}\,du = 2\int_0^{+1}\left(u^{3/2}-u^{1/2}\right)du = 2\left[\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right]_0^{+1}$$

$$= 2\left[\frac{2}{5}(1)-\frac{2}{3}(1) - \text{zeroes}\right] = \boxed{\frac{-8}{15}}$$

j) $\displaystyle\int_1^2 \frac{(\ln x)^2}{x}\,dx$ Very "famous" $\int u^n\,du$

where $u = \ln x$

$du = \dfrac{dx}{x}$

$$= \int_0^{\ln 2} u^2\,du$$

limits; $u_1 = \ln 1 = 0$

$u_2 = \ln 2$

$$= \frac{u^3}{3}\Big|_0^{\ln 2} = \frac{(\ln 2)^3}{3} - 0 = \boxed{\frac{(\ln 2)^3}{3}}$$

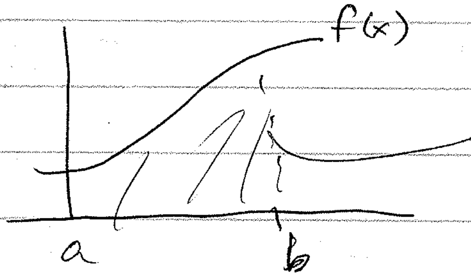
r) $\displaystyle\int_{-1}^{1}(x^2+7)^2\,dx$ Though not a $\int u\,du$, the binomial squared is easy to expand.

$$= \int_{-1}^{1}\left(x^4+14x^2+49\right)dx \quad \text{etc,}$$

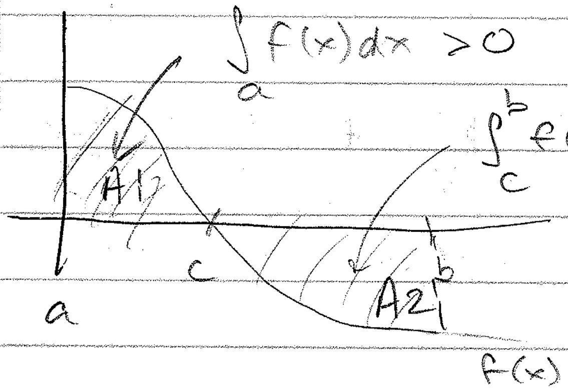
HW

Sec 34 - Area!

Notes



Area = $\int_a^b f(x)\,dx$ (actual area = signed or net area, since $f(x) > 0$ on $[a, b]$)



$\int_a^c f(x)\,dx > 0$

$\int_c^b f(x)\,dx < 0$

Actual area $= A_1 + A_2$

Net area $= A_1 - A_2$

Net area $= \int_a^b f(x)\,dx$

Actual area $= \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$

minus a negative integral

Reversing limits

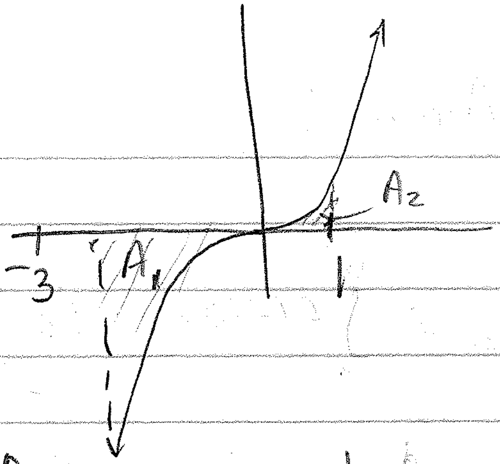
$= \int_a^c f(x)\,dx + \int_b^c f(x)\,dx$

or $= \left| \int_a^c f(x)\,dx \right| + \left| \int_c^b f(x)\,dx \right|$ ☆

Ex Find area between f(x) and x-axis on $[-3, 1]$ for $f(x) = x^3$

Solution The fcn. crosses the x-axis on the stated interval.

$A_1 = -\int_{-3}^{0} x^3\,dx \ , \ A_2 = \int_{0}^{1} x^3\,dx$



Actual area $= A_1 + A_2$

$$-\int_{-3}^{0} x^3\,dx + \int_{0}^{1} x^3\,dx = \int_{0}^{-3} x^3\,dx + \int_{0}^{1} x^3\,dx$$

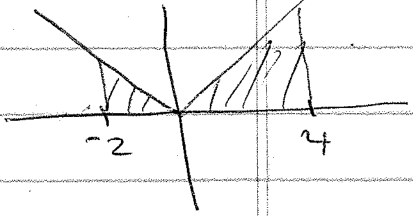
$$= \frac{x^4}{4}\Big|_{0}^{-3} + \frac{x^4}{4}\Big|_{0}^{1} = \frac{81}{4} + \frac{1}{4} = \boxed{\frac{82}{4}}$$

both > 0

$\downarrow$

$$\left|\frac{x^4}{4}\right|_{-3}^{0}$$

From HW # 2 $\int_{-2}^{4} |x|\,dx$



$$\text{Area} = \int_{-2}^{0} -x\,dx + \int_{0}^{4} x\,dx$$

$$= -\left(\frac{x^2}{2}\right)\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{4}$$

$$= 0 - \left(-\frac{2^2}{2}\right) + \frac{4^2}{2} - 0$$

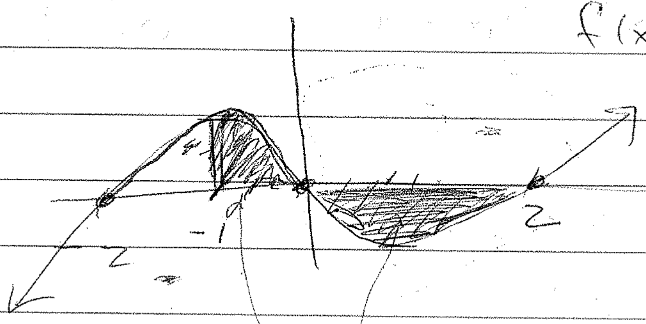
$$= 0 - -2 + 8 = \boxed{10}$$

Sec 34 con'd

#41 $f(x) = x^3 - 4x$ on $[-1, 2]$

Does the curve cross x-axis on the interval? If so, you must find actual area by splitting the integral into its pieces.

$f(x) = x(x^2 - 4) = 0$



Critical values: $f'(x) = 3x^2 - 4 = 0$

$x = \pm \frac{2}{\sqrt{3}}$

Are these max or min?

$f''(x) = 6x$, $f''(-2/\sqrt{3}) < 0$

so concave down, hence max; and $f''(2/\sqrt{3}) > 0$, so concave up, hence min. That showed us the graph's rough shape

abs value

$$\text{Area} = \int_{-1}^{0} (x^3 - 4x)\,dx + \left| \int_{0}^{2} (x^3 - 4x)\,dx \right|$$

$$= \frac{x^4}{4} - \frac{4x^2}{2}\Bigg|_{-1}^{0} + \left| \frac{x^4}{4} - \frac{4x^2}{2}\Bigg|_{0}^{2} \right|$$

$$= 0 - \left(\frac{(-1)^4}{4} - \frac{\overset{2}{\cancel{4}}(-1)^2}{\cancel{2}} \right) + \left| \left(\frac{16}{4} - 2(4) \right) \right|$$

$$= -\left(\frac{1}{4} - 2 \right) + \left| -4 \right|$$

$$= 1\,3/4 + 4 = \boxed{5\,3/4}$$

* Key is to use ( ) liberally. to avoid sign errors

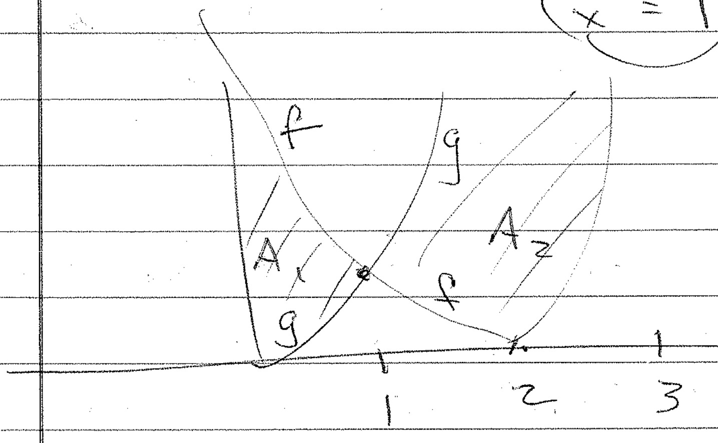
#5 $f(x) = x^2 - 4x + 4$ on $[0, 3]$

$g(x) = x^2$

Graphs intersect at ↗ $\cancel{x^2} - 4x + 4 = \cancel{x^2}$

$$-4x + 4 = 0$$

$\boxed{x = 1}$ intersection



$$\int_0^1 (f - g)\,dx + \int_1^3 (g - f)\,dx$$

$$\text{Area} = A_1 + A_2 = \int_0^1 f - g\,dx + \int_1^3 g - f\,dx$$

$$= \int_0^1 (\cancel{x^2} - 4x + 4 - \cancel{x^2})\,dx + \int_1^3 (x^2 - x^2 + 4x - 4)\,dx$$

$$= \int_0^1 (4 - 4x)\,dx + \int_1^3 (4x - 4)\,dx$$

$$= 4x - 2x^2 \Big|_0^1 + 2x^2 - 4x \Big|_1^3$$

$$= 4(1) - 2(1^2) - \text{zeroes}$$

$$+ 2(3^2) - 4(3) - \left(2(1^2) - 4(1)\right)$$

$$= 4 - 2 - 0 + 18 - 12 - (-2)$$

$$= 2 + 18 - 12 + 2 = \boxed{10}$$

Notice the liberal, tedious use of parentheses.

On this problem, both regions are above x-axis, so we didn't need to adjust by taking absolute value or changing limits of integration