HW Sec 2.4 Elasticity of Demand

#1. $q(p) = 60 - p$, $0 \le p \le 60$

a) $E(p) = -\frac{p}{q}\frac{dq}{dp} = \frac{-p}{60-p} \cdot -1 = \boxed{\frac{p}{60-p} = E(p)}$

b) $E(20) = \frac{20}{60-20} = \frac{20}{40} = \frac{1}{2} < 1$

@ $p=20$ Demand is inelastic (not "pulled" down sufficiently to offset revenue increase at price increase.)

c) $E(p) = \frac{p}{60-p} = 1$ when $p = 60 - p$ or $p = 30$

Thus, at $p = \$30$, elasticity is 1. A price increase or decrease will result in lower revenue, since

$E(p) = 1$ when $\frac{dR}{dp} = q(p)\left[1 - E(p)\right] = 0$

That is, when $p = 30$, $\frac{dR}{dp} = q(p)[1-1] = 0$, and so $R(30)$ is the maximum revenue for this demand fcn.

* Elasticity is a fcn. of price. Demand + price are related by the demand fcn. $q(p)$.
A commodity is elastic or inelastic to small changes in price depends on several factors

#3) $q(p) = 100 - \frac{p}{4}$

a) $E(p) = -\frac{p}{q} \cdot -\frac{1}{4} = \frac{p}{4q} = \frac{p}{4(100-p/4)} = \frac{p}{400-p}$

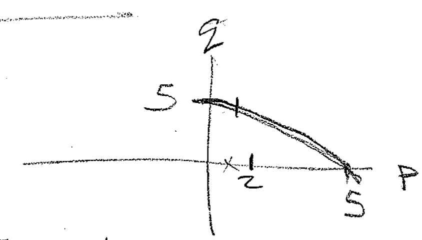
b) Find q at which $R(p)$ is maximized, i.e., at which $E(p) = 1$

$\frac{p}{400-p} = 1$ when $2p = 400$, $p = 200$;

thus, when $q(200) = 100 - \frac{200}{4} = 50$

The point to remember is that maximum revenue, ~~that is~~, the price at which $R'(p) = 0$, is that p where $E(p) = 1$.

$$\boxed{E(p) = 1 \Longleftrightarrow R'(p) = 0}$$



#4) $q(p) = \sqrt{25-p^2}$, $0 \le p \le 5$

p is currently $= \$2$, $q(2) = \sqrt{19} > 4$

Is demand at this price elastic or inelastic?

That is, would a small increase in price increase revenue or decrease revenue?

Elasticity for $E(p) = \frac{-p}{\sqrt{25-p^2}} \cdot \frac{1}{\cancel{2}}(25-p^2)^{-1/2}(-\cancel{2}p)$

$E(p) = \frac{p^2}{25-p^2}$, $E(2) = \frac{4}{21} < 1$, demand is inelastic at $p = \$2$

5. a) $q = 400 - .2p^2$, $\frac{dq}{dp} = -.4p$

$$E(p) = \frac{-p}{400-.2p^2} \cdot -.4p = \frac{.4p^2}{400-.2p^2}$$

$$E(20) = \frac{.4(20^2)}{400-.2(20^2)} = \frac{160}{320} = \frac{1}{2} < 1$$ demand inelastic at \$20

$$E(40) = \frac{.4(40^2)}{400-.2(40^2)} = \frac{640}{400-320} = \frac{640}{80} = 8 > 1$$ demand elastic at \$40

Thus, at \$20, revenue increases with a price increase.
At \$40, " decreases " " " "

b) $q = \frac{1000}{\sqrt{p}}$, $\frac{dq}{dp} = -500p^{-3/2}$, $E(p) = \frac{-p}{1000p^{-1/2}} \cdot -500p^{-3/2}$

$E(p) = \frac{1}{2} < 1$ at any p. Demand inelastic (revenue increases with price increase) at any price.

c) $q = \frac{500}{p^2}$, $\frac{dq}{dp} = \frac{-1000}{p^3}$, $E(p) = \frac{-p}{500/p^2} \cdot \frac{-1000}{p^3} = 2 > 1$

Demand is elastic at any price p.

d) $q = 625e^{-.025p}$, $\frac{dq}{dp} = 625e^{-.025p} \cdot (-.025) = -\frac{625}{40}e^{-.025p}$ $\quad -.025 = \frac{25}{1000} = \frac{1}{40}$

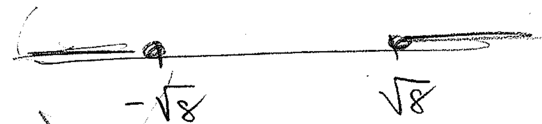
$$E(p) = \frac{-p}{\cancel{625e^{-.025p}}} \cdot \frac{-\cancel{625}}{40}\cancel{e^{-.025p}} = \frac{p}{40} \text{ or } .025p$$

$E(20) = \frac{20}{40} = \frac{1}{2} < 1$ demand inelastic; revenue increases for any increase in price

$E(40) = \frac{40}{40} = 1$ unit elasticity — this price gives max. revenue

#6) $q(p) = \sqrt{24-3p^2}$

a) Dom q: $24 - 3p^2 \geq 0 \rightarrow 8 - p^2 \geq 0 \rightarrow p^2 \geq 8$



$(-\infty, -\sqrt{8}) \cup (\sqrt{8}, \infty)$

But price ≥ 0 so Dom $= (\sqrt{8}, \infty)$

b) $E(p) = \dfrac{-p}{\sqrt{24-p^2}} \cdot \dfrac{1}{2}(24-3p^2)^{-1/2}(-6p) = \dfrac{3p^2}{24-p^2}$

c) Demand is inelastic when $\dfrac{3p^2}{24-p^2} < 1$

This algebra is more involved than you'll get on a test :-/

#7) $q(p) = (700-5p)^2$, $\dfrac{dq}{dp} = 10(700-5p)$ (p is in $\dfrac{1000 \text{ pesos}}{\text{liter}}$)

$E(p) = \dfrac{-p}{(700-5p)^{\cancel{2}}} \cdot -10(70\cancel{0}-5p) = \dfrac{10p}{700-5p}$; currently price $= 40,000$ pesos or $\boxed{p = 40}$

$E(40) = \dfrac{10(40)}{700-5(40)} = \dfrac{40\cancel{0}}{50\cancel{0}} < 1$ (demand inelastic at current price)

So to increase revenue, company could increase price.