Sec 22 Absolute max, min on closed interval

HW #3. $f(x) = x^3 - 6x^2 + 9x - 8$ on $[0,5]$

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 0$

$(x-3)(x-1) = 0$

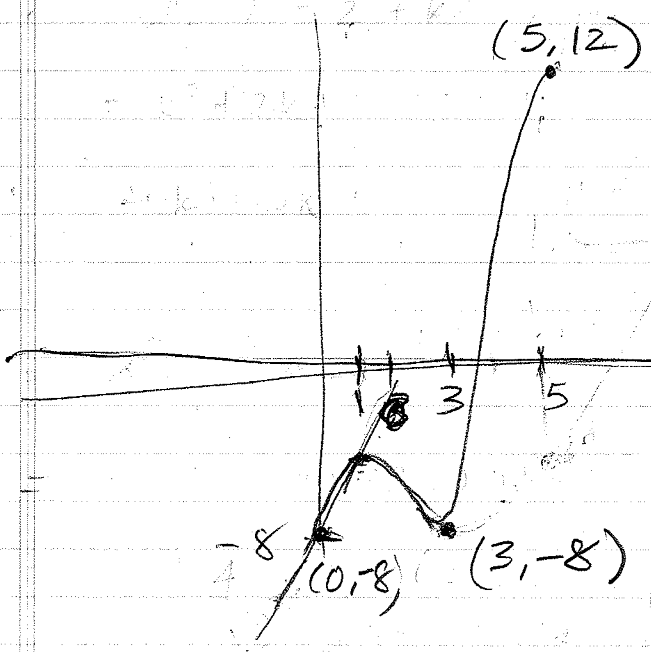
Crit. #s : $x = 3, 1$

$f(1) = 1 - 6 + 9 - 8 = -4$, $f(3) = \cancel{27} - \cancel{54} + \cancel{27} - 8$

$f(0) = -8$, $f(5) = 125 - 150 + 45 - 8 = 12$

Abs. min $(0,-8)$ and $(3,-8)$

Abs max $(5,12)$



On $[0,5]$ there are two x where $f(x)$ is the abs min $(0,-8)$ and $(3,-8)$

The abs max on $[0,5]$ is $(5,12)$

$f''(x) = 6x - 12$ $f''(1) = -6 < 0$ (c. down)

$f''(3) = 6 > 0$ (c. up)

So $f(1)$ is a local max, $f(3)$ local min

#4 $C(x) = 5(20-x)$ $R(x) = x(20-x)$

where, $20-x$ is quantity produced / sold

profit $P(x) = R(x) - C(x) = 20x - x^2 - 100 + 5x$

$P(x) = -x^2 + 25x - 100$

Max profit: $P'(x) = -2x + 25 = 0$ at $\boxed{x = \$12.50}$

#5 $P(x) = -x^3 + 9x^2 + 120x - 400$, $x \geq 5$

$P'(x) = -3x^2 + 18x + 120 = 0$

Max ~~sales~~ profit needs x items sold

$= -3(x^2 - 6x - 40) = 0$

$-3(x-10)(x+4) = 0$ $\boxed{x = 10}$

(discard $x = -4$)

Max profit in \$ into the original fcn.

$P(10) = -10^3 + 9 \cdot 10^2 + 120 \cdot 10 - 400$

$= -1000 + 900 + 1200 - 400$

$\boxed{= \$700,000}$

#6 $P(x) = -0.04x^2 + 240x - 10,000$

$P'(x) = -.08x + 240 = 0$ when $x = \frac{240}{.08}$

$x = \frac{24000}{8} = \boxed{3000 \text{ units}}$

#7 $P(x) = \ln(\underbrace{-x^3 + 3x^2 + 72x + 1}_{u})$, $x \in [0, 10]$

(i.e. [ln 1, ln ___]) range P(x)

range [0, ln ___]

$P'(x) = \frac{1}{u} \cdot du/dx$

$= \frac{-3x^2 + 6x + 72}{-x^3 + 3x^2 + 72x + 1} = 0$ when numerator $= 0$

$\boxed{x = 6}$

$= -3(x^2 - 2x - 24) = -3(x-6)(x+4) = 0$

#7 continued

a) $x = 6$ units sold will maximize profits

b) $P(6) = \ln(-6^3 + 3\cdot 6^2 + 72\cdot 6 + 1)$

$= \ln(-216 + 108 + 432 + 1)$

$= \underbrace{\ln(325)}_{\text{calculator needed}}$ thousands of dollars

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#4 analysis

Look again at #4. It's an easy problem once you determine the cost and revenue fcns.

x in this problem is price, and quantity is expressed as a fcn. of price, namely:

"At x dollars a piece, consumers will buy $20-x$ units of the item per day"

For example, at the following prices (x), the following demand/day is shown:

x (\$)	$20-x$ (quant)
0	20
1	19
2	18
3	17
4	16
5	15
6	14
7	13
8	12
9	11
10	10
⋮	
19	1
20	0

But the item costs \$5 to make, so she won't charge less than her cost.

But if she charges \$20, the demand $20-x=0$

The domain is thus $[5, 20]$

The cost fcn. is $\$5(20-x)$

The revenue fcn is $\underbrace{x}_{\text{price}}\underbrace{(20-x)}_{\text{quant}}$

Profit = Revenue − Cost

$\xi_j = \alpha^j + \beta^j$, α, β are roots of $x^2 - (k+1)x + 1 = 0$

$\alpha\beta = 1$, $\alpha + \beta = k+1$

- ξ_j are polys in k (of degree j)
- $c_0(\xi_j) = \overline{2, 1, -1,}$ $\overline{-2, -1, 1}$ (6-periodic)
- $\lambda_n := -\frac{1}{n}(\alpha^n + \beta^n + (-1)^n) = -\frac{1}{n}(\xi_n + (-1)^n)$

j	$c_0(\xi_j + (-1)^j)$	$c_0(\lambda_j)$
0	3	?
1	0	
2	0	0
3	-3	1
4	0	0
5	0	0
6	3	2
7	0	0

$$c_0(l_n) = -\sum_{d|n} (-1)^{n+n/d} \mu(d) \tfrac{1}{d} \cdot c_0(\lambda_{n/d})$$

$$\Rightarrow n\, c_0(l_n) = \sum_{d|n} (-1)^{n+n/d} \mu(d)\, c_0\left(-\tfrac{n}{d} \lambda_{n/d}\right)$$

$$= \sum_{d|n} (-1)^{n+n/d} \mu(d)\, c_0(\xi_{n/d} + (-1)^{n/d})$$

$$= \sum_{\substack{d|n \\ 3|n/d}} (-1)^{n+n/d} \mu(d)\, 3 (-1)^{\frac{n}{3d}}$$

$$= (-1)^n 3 \sum_{d|n,\, 3|n/d} \mu(d) = (-1)^n 3 \sum_{d|\frac{n}{3}} \mu(d) = \begin{cases} 0 \\ -3 \end{cases}$$

$$= \begin{cases} 0 & \text{if } 3 \nmid n \\ 0 & \text{if } n = 6, 9, \ldots \\ -3 & \text{if } n = 3 \end{cases}$$

$20x - x^2 = 5(20 - x)$

$0 = x^2 - 25x + 100$

$x = \dfrac{25 \pm \sqrt{625 - 4(100)}}{2}$

$= \dfrac{25 \pm \sqrt{225}}{2} = \dfrac{25 \pm 15}{2}$

$= \dfrac{10}{2}, \dfrac{40}{2}$

$= 5, 20$

$$l_4 = \dim L_4 = -\sum_{d|4} (-1)^{4/d} \mu(d) \frac{\lambda_{4/d}}{d} = -\left(\lambda_4 - \frac{\lambda_2}{2} - \frac{\lambda_1}{4}\right)$$

$$= -\left(-\tfrac{1}{4}(\alpha^4 + \beta^4 + 1)\right)$$

$\boxed{k \,|\, l_n}$ $n > 3$

$$n\, l_n = \sum_{d|n} (-1)^{n/d+n} \mu(d) (-\tfrac{n}{d} \cdot \lambda_{n/d})$$

$$= \sum_{d|n} (-1)^{n/d+n} \mu(d) \left(\alpha^{n/d} + \beta^{n/d} + (-1)^{n/d}\right) = (-1)^n \sum_{d|n} \mu(d) + \sum_{d|n} (-1)^{n/d+n} \mu(d) (\alpha^{n/d} + \beta^{n/d})$$

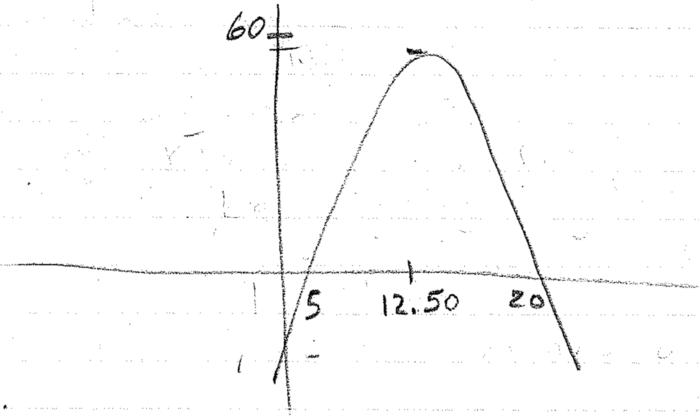
Con'd #4.

$$P(x) = (20-x)(x) - 5(20-x)$$
$$= 20x - x^2 - 100 + 5x$$

$$P(x) = -x^2 + 25x - 100 \quad \text{on } [5, 20]$$

$$P'(x) = -2x + 25 = 0 \quad \text{at } x = \$12.50$$

$P''(x) = -2$, which is negative for all x, so the graph is concave down everywhere, and $x = 12.50$ is clearly the price that maximizes profit. The parabola has its vertex at $(12.50, 56.25)$



(Max) profit is \$56.25/day when price is \$12.50.

Question: What quantity does this represent?

$(q = 20 - x)$ What does this actually mean?

Another question: How does the idea of max profit relate to marginal profit?

~~1) Anyons~~
~~2) RCFT~~
~~3) (fractional) Quantum Hall Effects~~
~~4) Edge modes~~
~~5) 1) + 4) +TQFT~~
~~6) Tunneling~~
~~7) Top entanglement entropy.~~