Related Rates HW - link

Given $C(x)$, dx/dt, and a value of x at which to evaluate the rate dC/dt, find dC/dt.

Scheme: $\dfrac{dC}{dt} = \dfrac{dC}{dx} \cdot \dfrac{dx}{dt}$

Given $C(x) = 15x^{4/3} + 54x^{2/3} + 600{,}000$

$$\frac{dC}{dx} = \frac{4}{3} \cdot 15x^{1/3} + \frac{2}{3} \cdot 54x^{-1/3} + 0$$

Marginal cost $\rightarrow$ $\dfrac{dC}{d\cancel{t}x} = 20x^{1/3} + 36x^{-1/3}$

Marginal cost is therefore given by $\dfrac{dC}{dx}$, that is, $C'(x)$.

For any # of printers in production x, the cost to produce the $x+1$ printer is found by $C'(x)$.

However, the question is to find the rate (variable t) at which cost is increasing each month at a given level of production, the level reached being $x = 1728$

Plugging what we know into the scheme:

$$\frac{dC}{dt} = \frac{dC}{dX} \cdot \frac{dx}{dt} \quad \text{for } x = 1728$$

$$\left.\frac{dC}{dt}\right|_{x=1728} = \left.\frac{dC}{dX}\right|_{x=1728} \cdot \frac{dX}{dt}$$

$$= \left(20\left(1728\right)^{1/3} + \frac{36}{(1728)^{1/3}}\right) \cdot 350$$

$$= (20 \cdot 12 + 36/12)(350)$$

$$= 85{,}050 \longrightarrow \$85{,}050/\text{month}$$