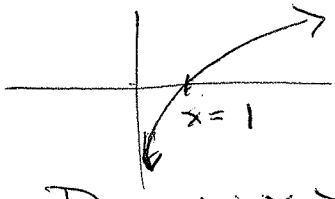
Formulas for Test 1

$\log x^r = r \log x$, $\log_a x = y \iff a^y = x$

$\log_b b = 1$, $\ln e = 1$,

$\log_b 1 = 0$, x-int $= 1$



Domain: $x > 0$

$y = mx + b$ $\quad (C(x) = mx + b)$

$y - y_1 = m(x - x_1)$ $\quad$ Eqn of line tangent to $f(x)$ at (x_1, y_1) Given values

$\downarrow$

$y - f(a) = f'(a)(x - a)$

$\hookrightarrow$ or $y = f'(a)(x-a) + f(a)$

where $f'(a) \equiv \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

$\hookrightarrow$ number

~~When does~~

In general, behavior of slope of tang.

is derived fcn $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\hookrightarrow$ eqn in x

Interest formulas

F, P, r, n, t

future value of, principal (loan, investment), r = rate of savings, lending per year "annual rate", n = # of compounding periods per year, t = # of years

When n is finite (annual ≡ simple $n=1$
semiannual $n=2$
monthly $n=12$
quarterly $n=4$
weekly $n=52$)

$$\boxed{F = P\left(1+\frac{r}{n}\right)^{nt}}$$

When n is "continuous", it doesn't appear in formula.

$$\boxed{F = Pe^{rt}}$$

(notice exponent has no "n", but has "r")

$e \approx 2.718$

Effective interest rate: $\boxed{\left(1+\frac{r}{n}\right)^{n} - 1}$

(It's a little bigger than r)

By the way; $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$

Needed to show cts. cmpding formula.

Solve for P, $F = Pe^{rt} \rightarrow \boxed{P = Fe^{-rt}}$

Solve for t, $F = Pe^{rt} \rightarrow \frac{F}{P} = e^{rt}$

$\rightarrow \ln F/P = rt$ the

Solve for t: $F = Pe^{rt}$

$$\frac{F}{P} = e^{rt}$$

Algorithm

$$\ln\left(\frac{F}{P}\right) = \ln e^{rt}$$

$$\ln\left(\frac{F}{P}\right) = rt \cancel{\ln e}^{\,1}$$

$$\boxed{\frac{\ln(F/P)}{r} = t}$$

To solve for "doubling time"

$$F = Pe^{rt} \rightarrow 2\cancel{P} = \cancel{P}e^{rt}$$

$$\rightarrow 2 = e^{rt} \rightarrow t = \frac{\ln 2}{r}$$

#7. Asking if $F = 1000$ when $t = 20$.

Given $r = .05$, $P = 300$

$$1000 \overset{?}{\leq} 300\, e^{(.05)(20)}$$

$$\begin{array}{r} 20 \\ .05 \\ \hline 1.0.0 \end{array}$$

$$3.33 = \frac{1000}{300} \overset{?}{\leq} e^{1} \approx 2.718$$

no - investment won't reach \$1000

i.e. $1000 > (2.718)(300)$

Set up #2

$P = 90{,}000$, $r = .04$, $n = 1$

find eff int rate

$$\left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.04}{1}\right)^1 - 1$$

$$= 1.04 - 1 = .04 \text{ eff int rate}$$

$$= r$$

$n = 4$ quarterly

$$\left(1 + \frac{.04}{4}\right)^4 - 1 = (1.01)^4 - 1$$