

Exam 1 Math 108 Fall 2016

1. Simplify as much as possible, writing answer w/o negative exponents.

a) $(3x^2z^{-4})^{-3} = 3^{-3}x^{-6}z^{12} = \frac{z^{12}}{3^3x^6} = \frac{z^{12}}{27x^6}$

b) $\frac{(-2xy^2)^3}{-(6xz^3)^2} = \frac{-8x^3y^6}{-36x^2z^6} = \frac{2x^{3-2}y^6}{9z^6} = \frac{2xy^6}{9z^6}$

2. Simplify each radical expression as much as possible, combining any "like terms".

a) $\sqrt[3]{-24x^5} = \sqrt[3]{-8 \cdot 3 \cdot x^3 \cdot x^2} = -2x\sqrt[3]{3x^2}$

b) $\sqrt{98} - \sqrt{50} - \sqrt{72} = \sqrt{2 \cdot 49} - \sqrt{2 \cdot 25} - \sqrt{2 \cdot 36}$
 $= 7\sqrt{2} - 5\sqrt{2} - 6\sqrt{2} = -4\sqrt{2}$

c) $\sqrt[4]{8a^3} \cdot \sqrt[4]{4a^2b} = \sqrt[4]{32a^3a^2b} = \sqrt[4]{16 \cdot 2a^5b}$
 $= 2\sqrt[4]{2a^4 \cdot ab} = 2a\sqrt[4]{2ab}$

3. Factor completely:

a) $35a^3b^2 + 14a^2b^3 = 7a^2b^2(5a + 2b)$

b) $12x^5 - 17x^4 + 6x^3 = x^3(12x^2 - 17x + 6)$

By "magic factoring": $(12)(6) = 72$; $-9 + -8 = -17$
 $(-9 \cdot -8 = 72)$

Rewrite $-17x$ as $-9x - 8x$:

$$\begin{aligned} x^3(12x^2 - 9x - 8x + 6) &= x^3(3x(4x - 3) - 2(4x - 3)) \\ &= x^3(4x - 3)(3x - 2) \end{aligned}$$

c) $x^2(x+1)^3 - 6x(x+1)^2 = x(x+1)^2[x(x+1) - 6]$
 $= x(x+1)(x^2 + x - 6) = x(x+1)(x+3)(x-2)$

4. Simplify the rational expressions as much as possible.
Specify any domain restrictions in answer.

a) $\frac{x^2 - 2x - 3}{2x + 4} \cdot \frac{x^2 - 4}{x^2 + 3x + 2} = \frac{(x-3)(x+1)(x-2)(x+2)}{2(x+2)(x+2)(x+1)}$

$$= \frac{(x-3)(x-2)}{2(x+2)}, \quad x \neq -1 \quad (\text{which is not apparent in final answer but was seen in original expression})$$

$$4(b) \quad \frac{x}{x^2+5x+6} - \frac{2}{x^2+3x+2} = \frac{x}{(x+2)(x+3)} - \frac{2}{(x+2)(x+1)}$$

$$\text{LCD} = (x+2)(x+3)(x+1)$$

$$\frac{x(x+1)}{\text{LCD}} - \frac{2(x+3)}{\cancel{x+2} \text{LCD}} = \frac{x^2 + x - 2x - 6}{\text{LCD}}$$

$$= \frac{x^2 - x - 6}{\text{LCD}} = \frac{(x-3)(x+2)}{(x+2)(x+3)(x+1)} = \frac{x-3}{(x+3)(x+1)}$$

Domain restriction $\rightarrow x \neq -2$

(no need to state obvious
natural domain of $x \neq -3, -1$)

5. Solve for x

$$a) x^3 - 3x^2 - 16x = -48$$

$$x^3 - 3x^2 - 16x + 48 = 0$$

$$(x^3 - 3x^2) - (16x - 48) = 0$$

$$x^2(x-3) - 16(x-3) = 0$$

$$(x-3)(x^2-16) = 0$$

$$(x-3)(x-4)(x+4) = 0$$

$$\boxed{x = 3, 4, -4}$$

$$b) \quad \frac{x+5}{x+3} + \frac{1}{x^2+2x-3} = 1$$

$$\frac{x+5}{x+3} + \frac{1}{\cancel{(x+3)(x-1)}} = 1$$

$$\frac{(x+5)(x-1)}{\text{LCD}} + \frac{1}{\text{LCD}} = 1$$

$$x^2 - x + 5x - 5$$

$$\frac{x^2 + 4x - 5}{\text{LCD}} = 1$$

Check if this is
in domain of
original problem.

If isn't, so
there's no solution.

$$x^2 + 4x - 5 = \text{LCD. cross mult.}$$

$$(x+5)(x+3)(x-1)(x+1)$$

$$x^2 + 4x - 5 = (x+3)(x-1) \quad (\text{the LCD})$$

$$x^2 + 4x - 5 = x^2 + 2x - 3$$

$$2x = 2, \quad x = 1$$

$$5c) \quad \sqrt{x+8} - \sqrt{x-4} = -2$$

$$\sqrt{x+8} = \sqrt{x-4} - 2 \quad \text{Best approach is to have one } \sqrt{\text{ per side.}}$$

$$(\sqrt{x+8})^2 = (\sqrt{x-4} - 2)^2$$

$$x+8 = x-4 - 4\sqrt{x-4} - 4$$

$$8 = -4\sqrt{x-4} - 8$$

$$16 = -4\sqrt{x-4}$$

$$-4 = \sqrt{x-4} \quad \text{no solution!} \quad \sqrt{a} \geq 0 \quad \text{must be}$$

6. Find domain of each.

$$a) \quad f(x) = \sqrt{2x+3} \quad 2x+3 \geq 0$$

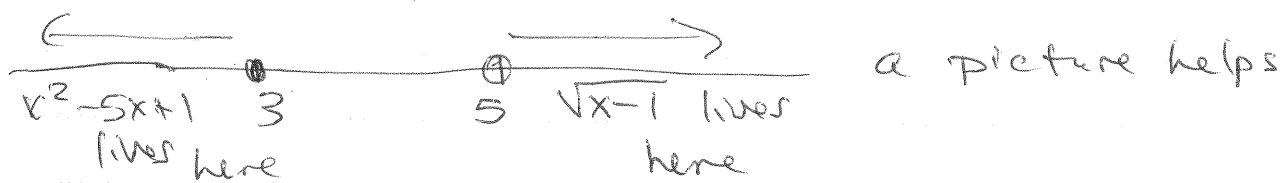
$$\begin{array}{|c|} \hline \text{Interval } [-3/2, \infty) \\ |x \geq -3/2| \\ \hline \end{array}$$

$$b) \quad g(x) = \frac{x+4}{2x^2-5x-12} = \frac{x+4}{(2x+3)(x-4)}$$

$$2x+3 \neq 0, x-4 \neq 0$$
 ~~$x \neq -3/2$~~

$$\boxed{x \neq -3/2} \quad \boxed{x \neq 4}$$

$$c) \quad h(x) = \begin{cases} x^2-5x+1, & \text{if } x \leq 3 \\ \sqrt{x-1}, & \text{if } x > 3 \end{cases}$$



a picture helps

So, simply read the restriction for each piece of the piecewise fn. That's the domain.

$$x \leq 3 \quad \text{or} \quad x > 5$$

Interval notation

$$(-\infty, 3] \cup (5, \infty)$$

7. Suppose $f(x) = \frac{3}{x-5}$, $g(x) = \sqrt{x-3}$

a) Evaluate (find the value of) $(f+g)(4)$:

$$(f+g)(x) = f(x) + g(x) = \frac{3}{x-5} + \sqrt{x-3} \quad (\text{simple!})$$

$$\text{So } (f+g)(4) = \frac{3}{4-5} + \sqrt{4-3} = -3 + 1 = -2$$

b) $f-g$ domain?

$$f(x) - g(x) = \frac{3}{x-5} - \sqrt{x-3}$$

where $x \neq 5$ and $x-3 \geq 0$ i.e. $x \geq 3$

Look at graph of this:

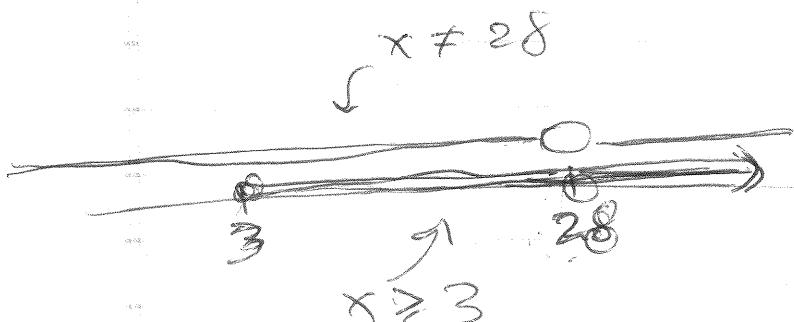


So $x \geq 3$ except $x \neq 5$

Interval notation: $[3, 5) \cup (5, \infty)$

c) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-3}) = \boxed{\frac{3}{\sqrt{x-3}-5}}$

d) domain $f \circ g$: $\sqrt{x-3} - 5 \neq 0$ and $x-3 \geq 0$
 $\sqrt{x-3} \neq 5$



$x-3 \neq 25$ and $x \geq 3$

~~(x ≠ 25)~~

$x \neq 28$ and $x \geq 3$

(must express as interval)

$[3, 28) \cup (28, \infty)$

"and" means intersection, so is where the two intersect

On 7d, don't be confused that answer has a union in it. The disjoint sets that resulted from the intersection is expressed as such. The algebraic $x \neq -8$ and $x \geq 3$ won't assure me that you see the "sln set", that is the domain.

$$8. \quad f(x) = \frac{x^3 - x}{x^2 + 4} \quad g(x) = \frac{2x+1}{x+4}$$

a) Is f odd, even, neither?

To determine this, write $f(-x)$ and decide if it is the same as $f(x)$ (~~then~~ hence even) or the same as $-f(x)$ (hence odd)

$$f(-x) = \frac{(-x)^3 - (-x)}{(-x)^2 + 4} = \frac{-x^3 + x}{x^2 + 4} = \frac{x - x^3}{x^2 + 4}$$

Definitely not same as $f(x)$
which is $\frac{x^3 - x}{x^2 + 4}$

Compare $\frac{-x^3 + x}{x^2 + 4}$ to $-\left(\frac{x^3 - x}{x^2 + 4}\right)$.

$$\frac{-x^3 + x}{x^2 + 4} \stackrel{?}{=} -\frac{-x^3 + x}{x^2 + 4} \quad \text{yes!}$$

$f(x)$ is odd //

b) Find x and y -intercepts

$x\text{-int: } f(x) = 0 \quad y\text{-int: } f(0) = \frac{0^3 - 0}{0^2 + 4}$

$$\frac{x^3 - x}{x^2 + 4} = 0$$

$$f(0) = 0$$

Just need $\rightarrow x^3 - x = 0$

$$x(x^2 - 1) = 0$$

$$x = 0 \text{ or } x = \pm 1$$

$\boxed{x\text{-ints: } (0; 0), (-1, 0), (1, 0)}$

$\boxed{(y\text{-int: } (0, 0))}$

c) Show g is one-one using definition.

1. Set $g(a) = g(b)$ for $a, b \in \text{dom } g$ the "suppose" step.
Suppose $g(a) = g(b)$
for a, b in g 's dom.

2. Plug these into g :

$$g(a) \Rightarrow \frac{2a-1}{a+4} = \frac{2b-1}{b+4} \Leftarrow g(b)$$

3. Simplify: Do not just cancel! Do the cross-multiply first.

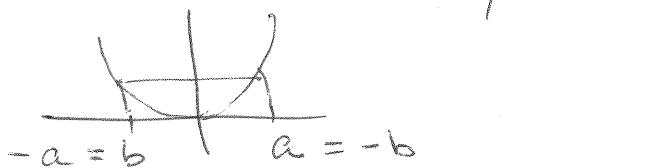
$$(2a-1)(b+4) = (a+4)(2b-1)$$

$$\cancel{2ab} - 8a - b - 4 = \cancel{2ab} - a + 8b - 4$$

$$-8a - b = -a + 8b$$

$$-7a = 7b \rightarrow \boxed{a = -b}$$

g is not one-one since the supposition that $g(a) = g(b)$ does not lead to $a = b$; it leads to $a = -b$ when $g(a) = g(b)$. (Like a parabola $y = x^2$)



d) $g^{-1}(x) = ?$

1. Write $y = \frac{2x-1}{x+4}$

2. Reverse x, y : $x = \frac{2y-1}{y+4}$

3. Solve for y in terms of x : $x(y+4) = 2y-1$

(Bring y terms together)

(Factor)

$$xy + 4x = 2y - 1$$

$$xy - 2y = -4x - 1$$

(Isolate y)

$$y(x-2) = -4x - 1$$

$$y = \frac{-4x-1}{x-2}$$

4.

$$\text{Write } y = g^{-1}(x) = \frac{-4x-1}{x-2}$$