Functions - Exponential Functions

Objective: Solve exponential equations by finding a common base.

As our study of algebra gets more advanced we begin to study more involved functions. One pair of inverse functions we will look at are exponential functions and logarithmic functions. Here we will look at exponential functions and then we will consider logarithmic functions in another lesson. Exponential functions are functions where the variable is in the exponent such as $f(x) = a^x$. (It is important not to confuse exponential functions with polynomial functions where the variable is in the base such as $f(x) = x^2$).

Solving exponetial equations cannot be done using the skill set we have seen in the past. For example, if $3^x = 9$, we cannot take the $x -$ root of 9 because we do not know what the index is and this doesn't get us any closer to finding x. However, we may notice that 9 is 3^2. We can then conclude that if $3^x = 3^2$ then $x = 2$. This is the process we will use to solve exponential functions. If we can re-write a problem so the bases match, then the exponents must also match.

Example 529.

$$
\begin{array}{rl}
5^{2x+1} = 125 & \text{Rewrite } 125 \text{ as } 5^3 \\
5^{2x+1} = 5^3 & \text{Same base, set exponents equal} \\
2x + 1 = 3 & \text{Solve} \\
\underline{-1 - 1} & \text{Subtract 1 from both sides} \\
2x = 2 & \text{Divide both sides by 2} \\
\overline{2} \;\; \overline{2} & \\
x = 1 & \text{Our Solution}
\end{array}
$$

Sometimes we may have to do work on both sides of the equation to get a common base. As we do so, we will use various exponent properties to help. First we will use the exponent property that states $(a^x)^y = a^{xy}$.

Example 530.

$$
\begin{array}{rl}
8^{3x} = 32 & \text{Rewrite 8 as } 2^3 \text{ and 32 as } 2^5 \\
(2^3)^{3x} = 2^5 & \text{Multiply exponents 3 and } 3x \\
2^{9x} = 2^5 & \text{Same base, set exponents equal} \\
9x = 5 & \text{Solve} \\
\overline{9} \;\; \overline{9} & \text{Divide both sides by 9} \\
x = \dfrac{5}{9} & \text{Our Solution}
\end{array}
$$

As we multiply exponents we may need to distribute if there are several terms involved.

Example 531.

$$
\begin{array}{rl}
27^{3x+5}=81^{4x+1} & \text{Rewrite 27 as } 3^3 \text{ and 81 as } 3^4\ (9^2 \text{ would not be same base)} \\
(3^3)^{3x+5}=(3^4)^{4x+1} & \text{Multiply exponents } 3(3x+5) \text{ and } 4(4x+1) \\
3^{9x+15}=3^{16x+4} & \text{Same base, set exponents equal} \\
9x+15=16x+4 & \text{Move variables to one side} \\
\underline{-9x \qquad -9x} & \text{Subtract } 9x \text{ from both sides} \\
15=7x+4 & \text{Subtract 4 from both sides} \\
\underline{-4 \qquad -4} & \\
11=7x & \text{Divide both sides by 7} \\
\overline{7} \quad \overline{7} & \\
\frac{11}{7}=x & \text{Our Solution}
\end{array}
$$

Another useful exponent property is that negative exponents will give us a reciprocal, $\frac{1}{a^n}=a^{-n}$

Example 532.

$$
\begin{array}{rl}
\left(\frac{1}{9}\right)^{2x}=3^{7x-1} & \text{Rewrite } \frac{1}{9} \text{ as } 3^{-2} \text{ (negative exponet to flip)} \\
(3^{-2})^{2x}=3^{7x-1} & \text{Multiply exponents } -2 \text{ and } 2x \\
3^{-4x}=3^{7x-1} & \text{Same base, set exponets equal} \\
-4x=7x-1 & \text{Subtract } 7x \text{ from both sides} \\
\underline{-7x \quad -7x} & \\
-11x=-1 & \text{Divide by } -11 \\
\overline{-11} \quad \overline{-11} & \\
x=\frac{1}{11} & \text{Our Solution}
\end{array}
$$

If we have several factors with the same base on one side of the equation we can add the exponents using the property that states $a^x a^y = a^{x+y}$.

Example 533.

$$
\begin{array}{rl}
5^{4x}\cdot 5^{2x-1}=5^{3x+11} & \text{Add exponents on left, combing like terms} \\
5^{6x-1}=5^{3x+11} & \text{Same base, set exponents equal} \\
6x-1=3x+11 & \text{Move variables to one sides} \\
\underline{-3x \qquad -3x} & \text{Subtract } 3x \text{ from both sides} \\
3x-1=11 & \text{Add 1 to both sides} \\
\underline{+1 \quad +1} & \\
3x=12 & \text{Divide both sides by 3} \\
\overline{3} \quad \overline{3} & \\
x=4 & \text{Our Solution}
\end{array}
$$

It may take a bit of practice to get use to knowing which base to use, but as we practice we will get much quicker at knowing which base to use. As we do so, we will use our exponent properties to help us simplify. Again, below are the properties we used to simplify.

$$(a^x)^y = a^{xy} \quad \text{and} \quad \frac{1}{a^n} = a^{-n} \quad \text{and} \quad a^x a^y = a^{x+y}$$

We could see all three properties used in the same problem as we get a common base. This is shown in the next example.

Example 534.

$$
\begin{array}{rl}
16^{2x-5}\cdot\left(\frac{1}{4}\right)^{3x+1}=32\cdot\left(\frac{1}{2}\right)^{x+3} & \text{Write with } a \text{ common base of } 2\\
(2^4)^{2x-5}\cdot(2^{-2})^{3x+1}=2^5\cdot(2^{-1})^{x+3} & \text{Multiply exponents, distributing as needed}\\
2^{8x-20}\cdot 2^{-6x-2}=2^5\cdot 2^{-x-3} & \text{Add exponents, combining like terms}\\
2^{2x-22}=2^{-x+2} & \text{Same base, set exponents equal}\\
2x-22=-x+2 & \text{Move variables to one side}\\
\underline{+x\qquad +x} & \text{Add } x \text{ to both sides}\\
3x-22=2 & \text{Add 22 to both sides}\\
\underline{+22+22} & \\
3x=24 & \text{Divide both sides by 3}\\
\overline{3}\quad\overline{3} & \\
x=8 & \text{Our Solution}
\end{array}
$$

All the problems we have solved here we were able to write with a common base. However, not all problems can be written with a common base, for example, $2 = 10^x$, we cannot write this problem with a common base. To solve problems like this we will need to use the inverse of an exponential function. The inverse is called a logarithmic function, which we will discuss in another secion.

Solve each equation.

1) $3^{1-2n}=3^{1-3n}$

2) $4^{2x}=\frac{1}{16}$

3) $4^{2a}=1$

4) $16^{-3p}=64^{-3p}$

5) $(\frac{1}{25})^{-k}=125^{-2k-2}$

6) $625^{-n-2}=\frac{1}{125}$

7) $6^{2m+1}=\frac{1}{36}$

8) $6^{2r-3}=6^{r-3}$

9) $6^{-3x}=36$

10) $5^{2n}=5^{-n}$

11) $64^b=2^5$

12) $216^{-3v}=36^{3v}$

13) $(\frac{1}{4})^x=16$

14) $27^{-2n-1}=9$

15) $4^{3a}=4^3$

16) $4^{-3v}=64$

17) $36^{3x}=216^{2x+1}$

18) $64^{x+2}=16$

19) $9^{2n+3}=243$

20) $16^{2k}=\frac{1}{64}$

21) $3^{3x-2}=3^{3x+1}$

22) $243^p=27^{-3p}$

23) $3^{-2x}=3^3$

24) $4^{2n}=4^{2-3n}$

25) $5^{m+2}=5^{-m}$

26) $625^{2x}=25$

27) $(\frac{1}{36})^{b-1}=216$

28) $216^{2n}=36$

29) $6^{2-2x}=6^2$

30) $(\frac{1}{4})^{3v-2}=64^{1-v}$

31) $4\cdot 2^{-3n-1}=\frac{1}{4}$

32) $\frac{216}{6^{-2a}}=6^{3a}$

33) $4^{3k-3}\cdot 4^{2-2k}=16^{-k}$

34) $32^{2p-2}\cdot 8^p=(\frac{1}{2})^{2p}$

35) $9^{-2x}\cdot(\frac{1}{243})^{3x}=243^{-x}$

36) $3^{2m}\cdot 3^{3m}=1$

37) $64^{n-2}\cdot 16^{n+2}=(\frac{1}{4})^{3n-1}$

38) $3^{2-x}\cdot 3^{3m}=1$

39) $5^{-3n-3}\cdot 5^{2n}=1$

40) $4^{3r}\cdot 4^{-3r}=\frac{1}{64}$