Logs + exponential fcns

Change of base formulas

① $\boxed{\log_a x = \dfrac{\log_b x}{\log_b a}}$ from $y = \log_a x$

$$a^y = x$$

$$\log_b a^y = \log_b x \quad \text{etc.}$$

e.g. $\log_3 x = \dfrac{\log x}{\log 3} = \dfrac{\ln x}{\ln 3} = \ldots$

② $\boxed{a^x = b^y \rightarrow y = x \log_b a}$

from $\log_b a^x = \log_b b^y$

$$x \log_b a = y \log_b b$$

$$\boxed{x \log_b x = y}$$

<u>Ex</u> Change $f(x) = 7^x$ to an equivalent fcn. in base 10; base e

$$7^x = 10^y \rightarrow y = x \log 7$$

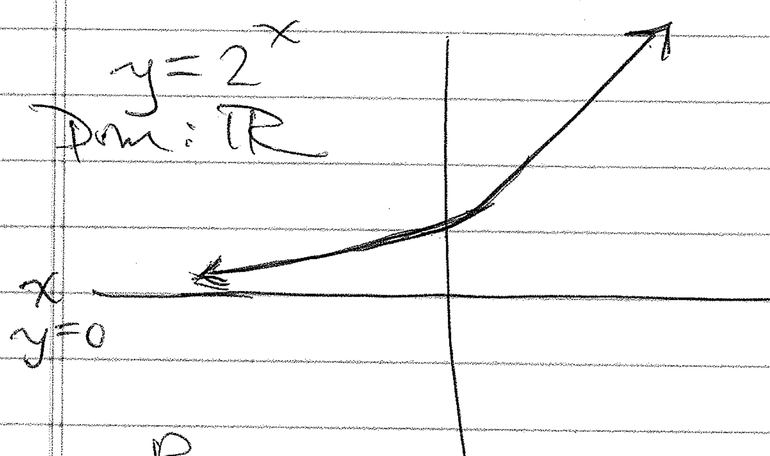
$$7^x = e^y \rightarrow y = x \ln 7$$

Given an exponential eqn like Ex 4.8 take the log of each side to a handy base using $\log_b b^x = x \cdot \log_b b = x \cdot 1 = x$

Growth that is not linear
but exponential. $2^{-1} = \frac{1}{2}$

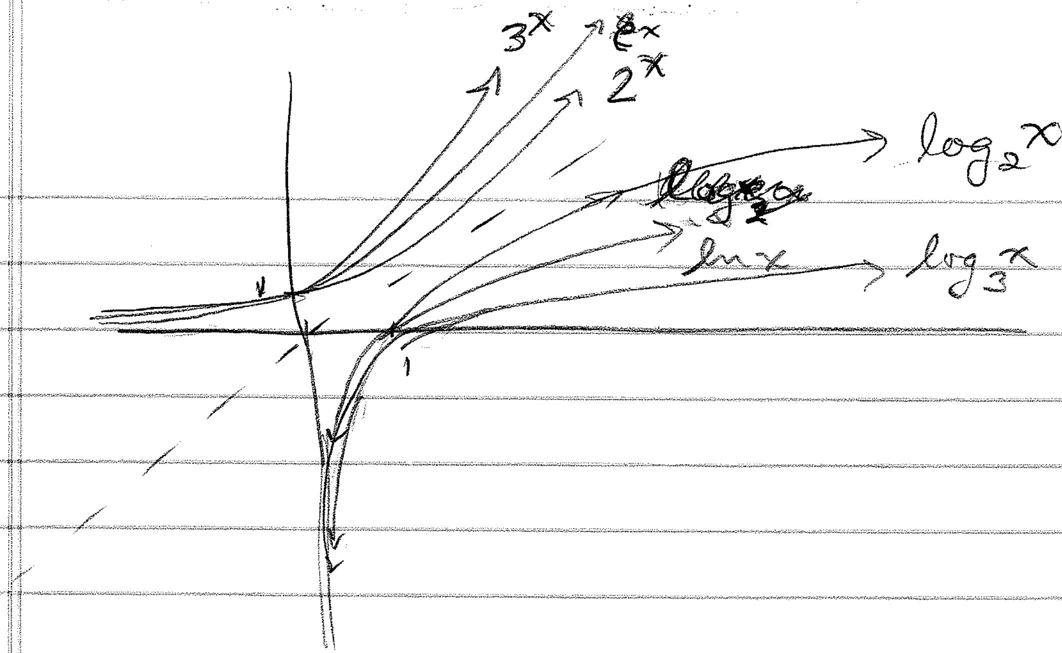
Say $y = 2^x$ is the inverse fcn. of $y = \log_2 x$.

$y = 2^x$
Dom: $\mathbb{R}$



x	y
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
6	64

Range: $y > 0$
Dom: $x \in \mathbb{R}$



$x \in \mathbb{R}$ so what is meant by 2^{π}, for ex.
Since we cannot calculate a power that is non-repeating, non-terminating, we have to look at the graph for inspiration. The curve of $\log x$ is "continuous," which means there are no holes, asymptotes on the stated domain ($\mathbb{R}$ is this case)

Also, it is "smooth" which means it is continuous plus it has no sharp pts. Using limit theory, we determine that as x approaches π, $2^x \to 2^{\pi}$
There is a point on the graph of 2^x at $x = \pi$ so 2^x is defined for this irrational value of x. In fact, this is the case for all $x \in \mathbb{R}$.

This is not a trivial matter. $y = b^x$ is defined for all $x \in \mathbb{R}$, even though b^x is not exactly computable.

$$\boxed{y = a^x} \quad x \in \mathbb{R}$$

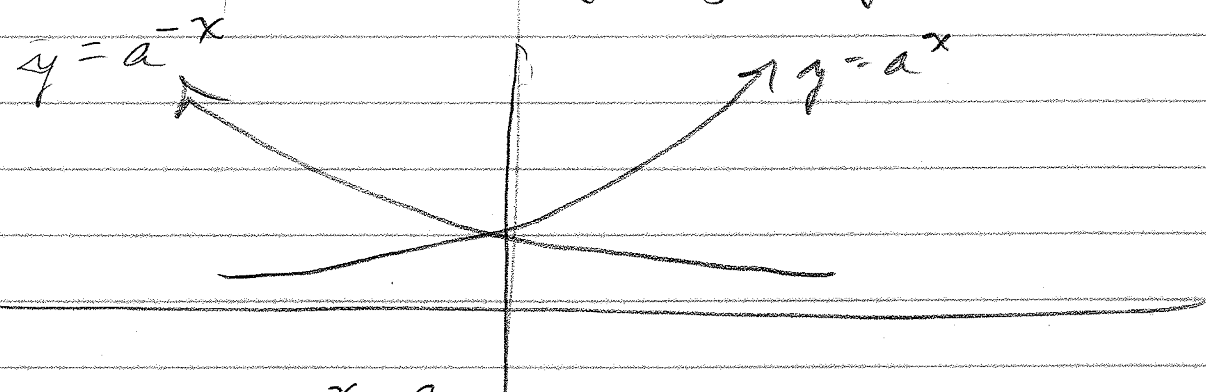
$0 < a < 1$, say $a = 1/10$

x	$\frac{1}{10}^x$
-2	$(1/10)^{-2} = 10^2$
-1	$(1/10)^{-1} = 10$
0	$(1/10)^0 = 1$
1	$(1/10)^1 = 1/10$
2	$(1/10)^2 = 1/100$

$a > 1$, say $a = 10$

x	10^x
-2	$10^{-2} = 1/100$
-1	$10^{-1} = 1/10$
0	$10^0 = 1$
1	$10^1 = 10$
2	$10^2 = 100$

In general, the reflected graphs $y = a^x$ and $y = a^{-x}$ inform the curve sketching of $y = a^x$, $0 < a < 1$



So given $y = a^x$ for $0 < a < 1$, just reflect the graph of $y = a^x$ for $a > 1$

$$y = \left(\frac{1}{a}\right)^x = a^{-x}$$

We'll usually deal with $a > 1$ + $y = a^{-x}$ will cover the bases that are less than 1

Solving exponential eqns.

What is x when $2^x = 8$? Clearly, $x = 3$
You can compute this from your knowledge of powers of 2 and see it on the graph.

Another, $2^{3x+1} = \sqrt{2}$ (book)
Recognize the power of 2 that $\sqrt{2}$ represents.

|| **Property** If $a^x = a^y$ then $x = y$ and vice versa. ||

In mathematics this sort of statement is written compactly as:

$$a^x = a^y \text{ if and only if } x = y$$

OR

$$\boxed{a^x = a^y \iff x = y}$$

We'll use this to solve eqns. like

$$2^{3x+1} = \sqrt{2} = 2^{\frac{1}{2}}$$

So $3x + 1 = \frac{1}{2}$