~~Ex~~ Exam 4

1a) $-130^\circ + 360^\circ = 230^\circ$ coterminals

$-130^\circ - 360^\circ = -490^\circ$

b) $\frac{11\pi}{6} + 2\pi = \frac{23\pi}{6}$ coterminals

$\frac{11\pi}{6} - 2\pi = \frac{-\pi}{6}$

2) a) $90 - 57 = 33^\circ$ complement

$180 - 57 = 123^\circ$ supplement

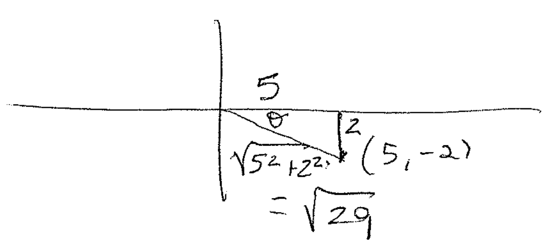
b) $\frac{\pi}{2} - \frac{2\pi}{5} = \frac{5\pi - 4\pi}{10} = \frac{\pi}{10}$ comp.

$\pi - \frac{2\pi}{5} = \frac{3\pi}{5}$ supp

3) $115^\circ \times \frac{\pi}{180^\circ} = \frac{115\pi}{180}$ rad $= \frac{23\pi}{36}$

$4 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = \frac{720^\circ}{\pi \text{ rad}}$ (Alternate: $\approx 4 \times \frac{57.3^\circ}{\text{rad}}$)

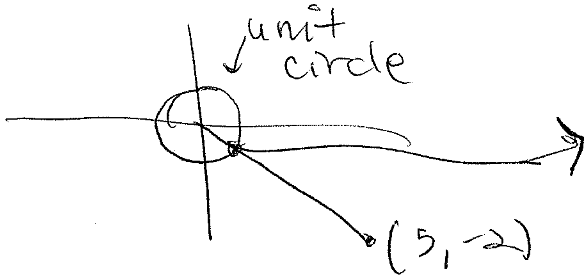
4)



$\sin\theta = \frac{-2}{\sqrt{29}}$ $\sec\theta = \frac{\sqrt{29}}{5}$

$\cos\theta = \frac{5}{\sqrt{29}}$ $\csc\theta = -\frac{\sqrt{29}}{2}$

$\tan\theta = -\frac{2}{5}$ $\cot\theta = \frac{-5}{2}$



$\left(\frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}}\right)$

5. a) Range $[1, 5]$ → Means midline is $y = 3$

Amp $A = 2 = \frac{5-1}{2}$

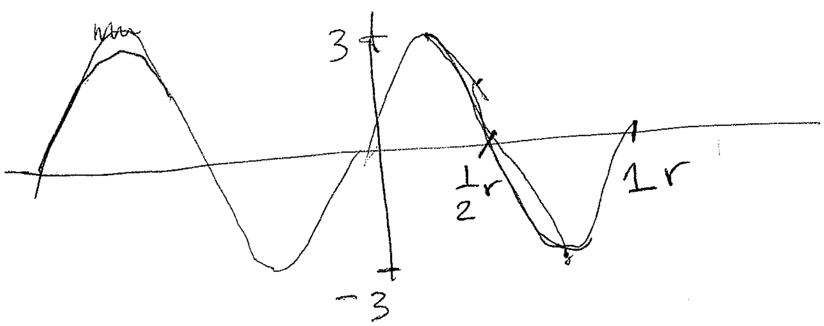
Period $= 8\pi = \frac{2\pi}{B}$ and so $B = \frac{2\pi}{8\pi} = \frac{1}{4}$

Reflect ~~cosθ~~ through x-axis (negate entire fn.)

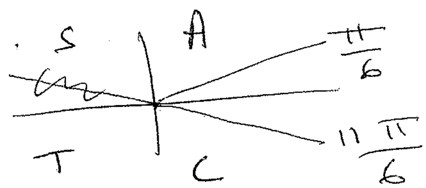
$y = -A\cos(Bx) = -2\cos\frac{1}{4}x$

6. $f(x) = 3\sin(2\pi x)$ $A = 3$, $B = 2\pi$

Period $= \frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1^r$

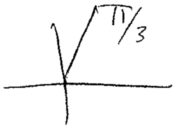


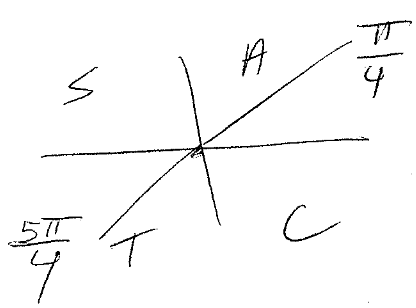
7. • $\cos\pi = 0$



• $\cos\frac{11\pi}{6} = \oplus\cos\frac{\pi}{6} = +\frac{\sqrt{3}}{2}$

• $\sin\frac{\pi}{3} = \frac{1}{2}$



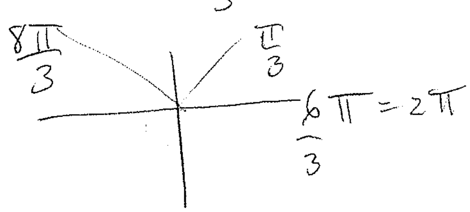


• $\sin\frac{5\pi}{4} = \ominus\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

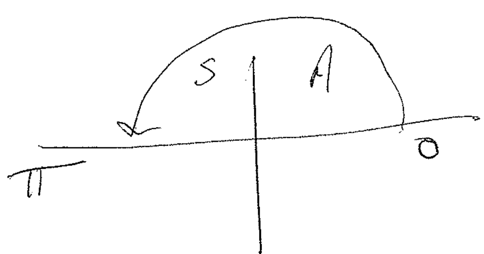
coterminal

• $\tan\frac{8\pi}{3} = \tan\frac{2\pi}{3} = -\tan\frac{\pi}{3} = -\frac{\sqrt{3}}{1}$

reference

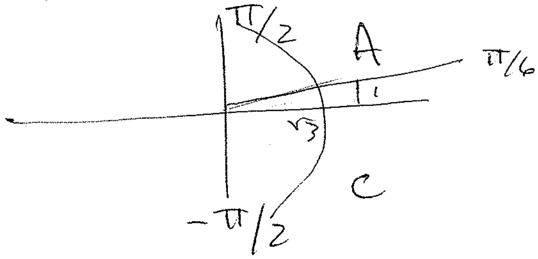


8. $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

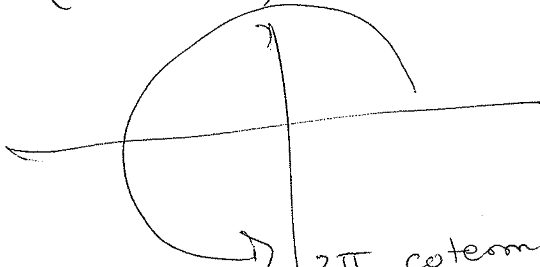


Has to be in range $[0, \pi]$

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

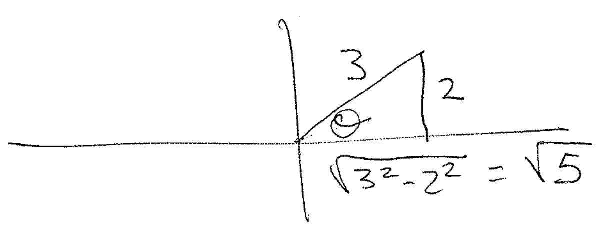


$\arcsin\left(\sin \frac{3\pi}{2}\right) = \arcsin\left(\sin\left(-\frac{\pi}{2}\right)\right) = -\frac{\pi}{2}$



$\frac{3\pi}{2}$ coterminal w/ $-\frac{\pi}{2}$ in range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

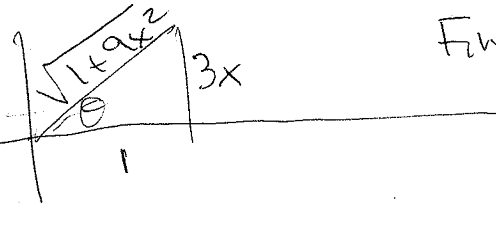
$\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \sqrt{5}/3$



$\sec\left(\arctan(3x)\right) = ?$, Draw the triangle with $\frac{opp}{adj} = \frac{3x}{1}$

Find hypot.

$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{1+9x^2}}{1}$



9. $$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \stackrel{?}{=} 2\csc\theta$$

LCD: $$\frac{\sin\theta\cdot\sin\theta + (1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{\overbrace{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}^{1}}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{2+2\cos\theta}{1+\cos\theta}\cdot\frac{1}{\sin\theta}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\cancel{(1+\cos\theta)}}\cdot\frac{1}{\sin\theta} = \frac{2}{\sin\theta} = 2\csc\theta \checkmark$$

10. $$\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta} \stackrel{?}{=} \cot\alpha\cot\beta + 1$$

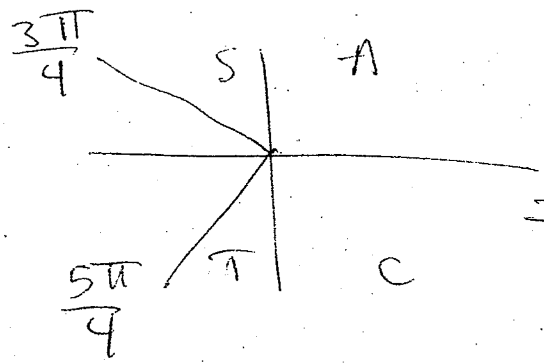
$$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta} = \frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}$$

$$= \cot\alpha\cot\beta + 1 \checkmark$$

11. $2\cos(5x) = -\sqrt{2}$ Find all solns.

$$\cos(5x) = -\frac{\sqrt{2}}{2} = \frac{-1}{\sqrt{2}}$$

(Go $\pm 2\pi n$ for each distinct soln. found)



$$\Rightarrow 5x = \frac{3\pi}{4} \pm 2\pi n$$

or $5x = \frac{3\pi}{4}, \frac{11\pi}{4}, \frac{19\pi}{4}, \ldots$

so $x = \frac{3\pi}{20}, \frac{11\pi}{20}, \frac{19\pi}{20}, \ldots$

AND $5x = \frac{5\pi}{4}, \frac{13\pi}{4}, \frac{21\pi}{4}, \ldots$

so $x = \frac{5\pi}{20}, \frac{13\pi}{20}, \frac{21\pi}{20}, \ldots$

You still need negative coterminals

$$x = \frac{3\pi}{20} \pm 2\pi n$$

&

$$\frac{5\pi}{20} \pm 2\pi n$$

12) $1 + \sin x = 2\cos^2 x$ on $[0, 2\pi)$

$$0 = 2\cos^2 x - \sin x - 1$$

$$0 = 2(1 - \sin^2 x) - \sin x - 1$$

$$0 = 2 - 2\sin^2 x - \sin x - 1$$

$$0 = 1 - 2\sin^2 x - \sin x$$

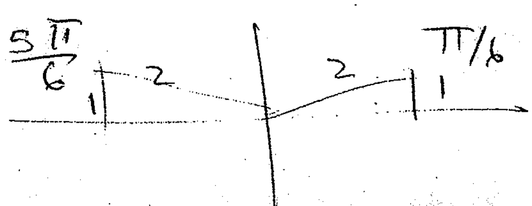
$$0 = -2\sin^2 x - \sin x + 1$$

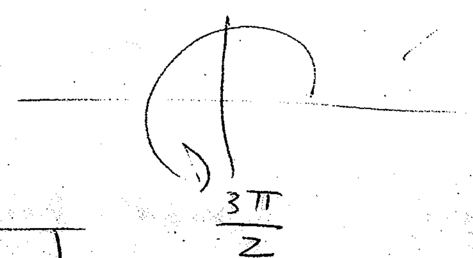
$$0 = 2\sin^2 x + \sin x - 1$$

$$0 = (2\sin x - 1)(\sin x + 1)$$

$$2\sin x = 1 \qquad \sin x = -1$$

$$\sin x = \frac{1}{2}$$



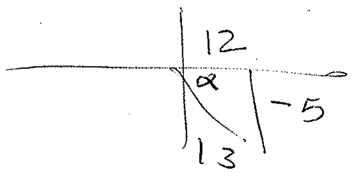


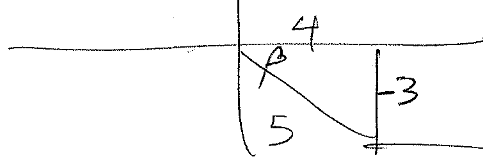
$$\boxed{x = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{3\pi}{2}}$$

13) $\sin\alpha = -5/13$ $\quad \frac{3\pi}{2} < \alpha < 2\pi$ $\qquad$ $\cos\beta = 4/5$ $\quad \frac{3\pi}{2} < \beta < 2\pi$ $\qquad$ Q IV

Note: $3\pi < 2\alpha < 4\pi$ $\qquad 3\pi < 2\beta < 4\pi$

and $\frac{3\pi}{2} < \frac{\alpha}{2} < \pi$ $\qquad \frac{3\pi}{4} < \frac{\beta}{2} < \pi \rightarrow$ Q II $\quad \rightarrow$ For parts (f), (h)





b) $\cos\alpha = \boxed{12/13}$ $\qquad$ a) $\sin\beta = \boxed{-3/5}$

c) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{12}{13}\cdot\frac{4}{5} - \frac{-5}{13}\cdot -\frac{3}{5}$

$= \frac{48-15}{65} = \boxed{\frac{33}{65}} > 0$ $\quad$ so Q I or IV

d) $\alpha+\beta$ is in Q I or IV from part c

$\frac{3\pi}{2}+\frac{3\pi}{2} < \alpha+\beta < 2\pi + 2\pi$

$3\pi < \alpha+\beta < 4\pi \longrightarrow \boxed{\text{Q IV}}$

e) $\sin(2\alpha) = 2\sin\alpha\cos\alpha = 2\left(-\frac{5}{13}\right)\left(\frac{12}{13}\right) = -\frac{120}{169} < 0$

so in Q III or IV

f) 2α is in $\boxed{\text{Q IV}}$

since $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 > 0$

in Q IV

h) $\frac{\beta}{2}$ in Q $\overline{\text{II}}$ (see note)

g) $\cos\left(\frac{\beta}{2}\right) = \sqrt{\frac{1+\cos\beta}{2}} = \sqrt{\frac{1+4/5}{2}} = \sqrt{9/10}$