

# Exam 3

1a)  $2 - 3x \leq 5(2x + 3)$

$$2 - 3x \leq 10x + 15$$

$$-13x \leq 13$$

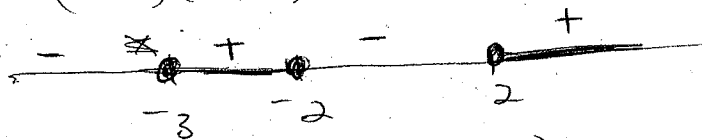
$$x \geq -1 \quad [-1, \infty)$$

b)  $x^3 + 3x^2 - 4x - 12 \geq 0$

$$x^2(x+3) - 4(x+3) \geq 0$$

$$(x^2 - 4)(x+3) \geq 0$$

$$(x-2)(x+2)(x+3) \geq 0$$



$$[-3, -2] \cup [2, \infty)$$

c)  $|x-1| > 4$

$$x-1 > 4 \text{ or } x-1 < -4$$

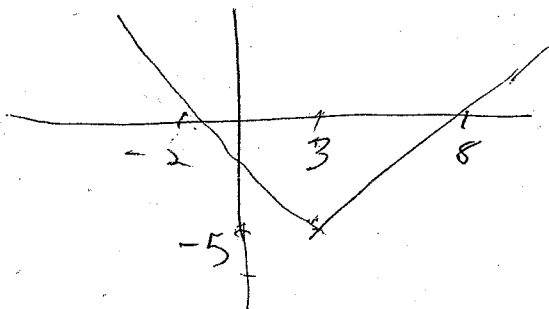
$$x > 5 \text{ or } x < -3 \quad (-\infty, -3) \cup (5, \infty)$$

d)  $\frac{3x+2}{x-2} \leq 5 \rightarrow \frac{3x+2}{x-2} - 5 \leq 0 \rightarrow \frac{3x+2-5x+10}{x-2} \leq 0$

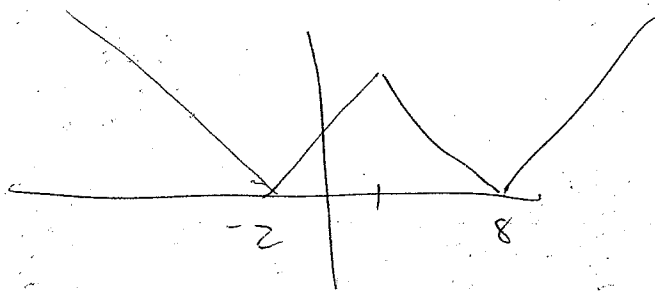
$$\rightarrow \frac{-2x+12}{x-2} \leq 0 \quad \text{Number line with roots at 2 and 6. Sign is positive between 2 and 6, negative elsewhere. Solution: } (-\infty, 2) \cup [6, \infty)$$

2)  $f(x) = |x-3| - 5 = 0$  (x-intercepts)

$$\rightarrow |x-3| = 5 \rightarrow x-3 = 5 \text{ or } x-3 = -5$$
$$x = 8 \text{ or } x = -2$$



Graph of  $f(x)$



Graph  $|f(x)| = g(x)$

(Make negative y portion positive)

Exam 3

Test 3 con'd

3.) a) Solve system  $\begin{cases} -7x + 7y = -21 \\ 4x - 2y = 22 \end{cases} \rightarrow \begin{cases} -7x + 7y = -21 \\ 2x - y = 11 \end{cases}$

$\rightarrow$   $\begin{array}{r} -7x + 7y = -21 \\ 14x - 7y = 77 \\ \hline 7x = 56 \\ x = 8 \end{array}$   $\rightarrow$   $\begin{array}{r} 4(8) - 2y = 22 \\ -2y = -10 \\ y = 5 \end{array}$

Ans  
 $(8, 5)$

b)  $\begin{cases} ① x^2 + y = 0 \\ ② y = 2x^2 + 3x - 6 \end{cases}$

Subst.  $y = -x^2$  into ②

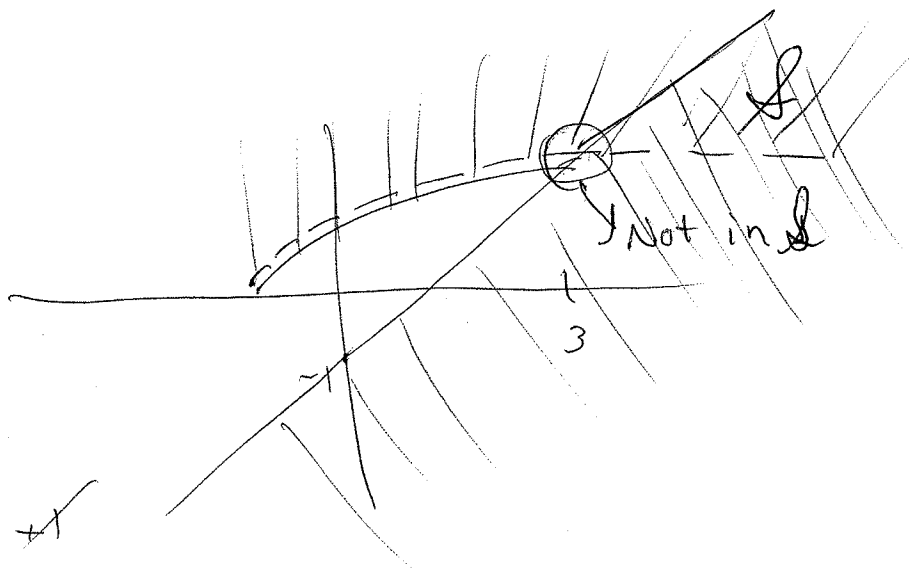
$-x^2 = 2x^2 + 3x - 6$   
 $0 = 3x^2 + 3x - 6$   
 $0 = x^2 + x - 2 = (x-1)(x+2)$   
 $x = 1, -2$   
 $y = -1, -4$

$(1, -1)$  &  $(-2, -4)$

$x^2 + y = 0$   
 $y = -1$

$(-2)^2 + y = 0$   
 $y = -4$

4)  $\begin{cases} y > \sqrt{x+1} \\ y \leq x-1 \end{cases}$



$\sqrt{x+1} = x-1$   
 $x+1 = (x-1)^2$   
 $x+1 = x^2 - 2x + 1$   
 $0 = x^2 - 3x = x(x-3)$

$x = 0, x = 3, y = 3-1 = 2$

$\uparrow$   
not a vertex

$(3, 2)$  vertex, not in  $S$

$$5. a) \log_5 1 = 0 \quad (5^0 = 1)$$

$$b) \log_3 9 = 2 \quad (3^2 = 9)$$

$$c) \log_{16} 4 = 1/2 \quad (16^{1/2} = \sqrt{16} = 4)$$

$$d) \log_4 \frac{1}{64} = \log_4 64^{-1} = -1 \log_4 64 = -1(3) = -3$$

$$e) e^{\ln 42} = e^{\log_e 42} = 42$$

$$4^3 = 64$$

by property  $a^{\log_a x} = x$

$$f) \log_5 75 - \log_5 3 = \log_5 \left( \frac{75}{3} \right) = \log_5 25 = 2$$

$$6. a) \quad 3 \log_3 x - 4 \log_3 y + \log_3 z = \log_3 x^3 - \log_3 y^4 + \log_3 z$$
$$= \log_3 \left( \frac{x^3 z}{y^4} \right) \text{ by properties, } \underline{\text{known!}}$$

$$c) \log_b 12 = \frac{\ln 12}{\ln b} \quad \text{by } \log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b} = \text{etc...}$$

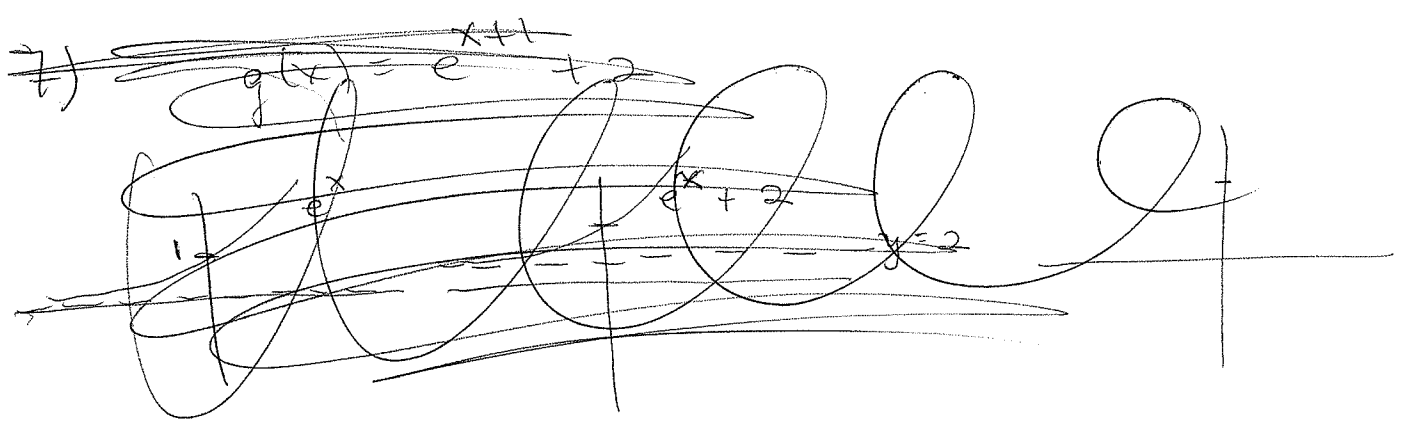
$$b) \log_2 4x^2 y = \log_2 4 + \log_2 x^2 + \log_2 y$$
$$= 2 + 2 \log_2 x + \log_2 y$$

$$d) \log_3 25 \approx x? \quad 3^2 = 9, \quad 3^3 = 27$$

so  $x$  is much closer to 3 than 2.

say  $x \approx 2.8$  or higher, less than 3

$$\boxed{2.8 \leq x < 3}$$



8) c)  $\log(x+2) = 3$

$10^3 = x+2 \rightarrow \boxed{x=998}$

d)  $\log_2(x^2 - 6x) - \log_2(1-x) = 3$

$$\log_2\left(\frac{x^2 - 6x}{1-x}\right) = 3$$

$$2^3 = \frac{x^2 - 6x}{1-x}$$

$$0 = \frac{x^2 - 6x}{1-x} - 8 = \frac{x^2 - 6x - 8(1-x)}{1-x}$$

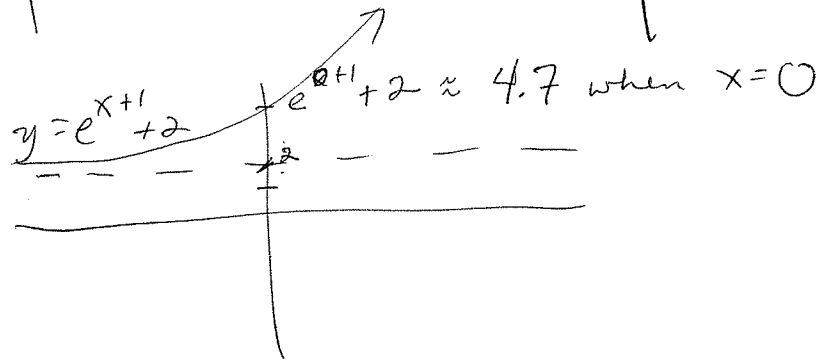
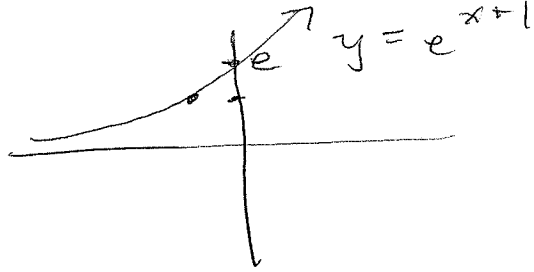
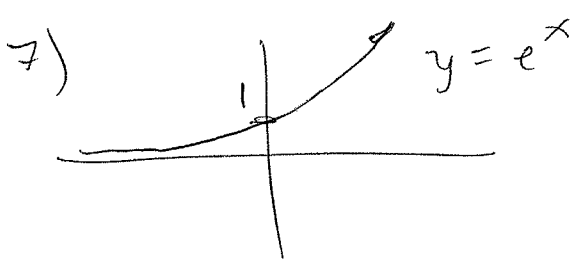
$$\text{So } 0 = x^2 + 2x - 8$$

$$0 = (x-2)(x+4)$$

$x = 2, -4$ , but  $x \neq 2$  since

$\log_2(1-2) \text{ DNE}$   
 $\uparrow$   
 negative

$$\boxed{x = -4}$$



8) a)  $4^{x+3} = 8^{x-1} \rightarrow (2^2)^{x+3} = (2^3)^{x-1}$

~~2^{2x+3}~~  $2^{2(x+3)} = 2^{3(x-1)}$

so  $2(x+3) = 3(x-1)$

$x=9$

b)  $5^{x+3} = 6^{x-1}$

$\log_5 5^{x+3} = \log_5 6^{x-1}$

$(x+3) \log_5 5 = \dots$

$x+3 = (x-1) \log_5 6$

$x+3 = x \log_5 6 - \log_5 6$

$x - x \log_5 6 = -3 - \log_5 6$

$x(1 - \log_5 6) = -3 - \log_5 6$

$x = \frac{-3 - \log_5 6}{1 - \log_5 6}$

or  $\log_6 5^{x+3} = \log_6 6^{x-1}$

Likewise,  
answer will  
appear different