More Even and Odd Function Practice

Math 130 *Kovitz*

In problems 1. through 11.: Decide whether the function f with the given rule is even, odd, or neither. Justify your answer.

1. $f(x) = 1/x$
2. $f(x) = (x^2+4)(x-2)(x+2)$
3. $$f(x) = \begin{cases} 5x+4 & \text{if } x > 0 \\ 5x-4 & \text{if } x < 0 \end{cases}$$
4. $f(x) = \dfrac{1-x}{1+x} + \dfrac{1+x}{1-x}.$
5. $f(x) = \dfrac{1-x}{1+x} - \dfrac{1+x}{1-x}.$
6. $f(x) = \dfrac{x-1}{x}.$
7. $f(x) = x - \dfrac{1}{x}$
8. $f(x) = |x|$
9. $f(x) = \sqrt{|x|}$
10. $f(x) = \dfrac{x^2-4x+4}{x}$
11. $f(x) = |x|/x$
12. $f(x) = \dfrac{x^3-1}{x-1}.$

For each of the following problems, decide whether the solutions to the equation constitute an odd function, an even function, neither, or both.

13. $x^4 = y^4$
14. $x^2 + y^2 = 0$, considering the solutions over the real numbers only.
15. $x^2 + y^2 = 1$ with $y \geq 0$.

Answers below

Answers with Justifications

1. Odd. For all a: $f(-a) = 1/(-a)$ and $-f(a) = -1/a$. They are equal.
2. Even. It reduces to $x^4 + 16$, which is even by the rule of even powers.
3. Odd. If $a > 0$: $f(a) = 5a + 4$ and $f(-a) = 5(-a) - 4 = -5a - 4 = -(5a+4) = -f(a)$. If $a < 0$: $f(a) = 5a - 4$ and $f(-a) = 5(-a) + 4 = -5a + 4 = -(5a - 4) = -f(a)$.

   It is much easier to look at the graph and note that it is symmetric through the origin.
4. Even. It simplifies to $f(x) = \dfrac{2(1+x^2)}{1-x^2}$, so it's even by the rule of even powers.
5. Odd. It simplifies to $f(x) = \dfrac{-4x}{1-x^2}$, so $f(a) = \dfrac{-4a}{1-a^2}$ and $f(-a) = \dfrac{4a}{1-a^2} = -f(a)$.
6. Neither. Because $f(1) = 0$ and $f(-1) = 2$, it cannot possubly be odd or even.
7. Odd. $f(-a) = -a + \dfrac{1}{a} = -\left(a - \dfrac{1}{a}\right)$.
8. Even. $f(-a) = |-a| = |-1||a| = |a|$.
9. Even. $f(-a) = \sqrt{|-a|} = \sqrt{|-1||a|} = \sqrt{|a|}$.
10. Neither. $f(2) = 0$ but $f(-2) = -8$. Simplifying the numerator to $(x-2)^2$ does not change this fact.
11. Odd. $f(-a) = |-a|/(-a) = -|a|/a = -(|a|/a) = -f(a)$.
12. Neither. $f(2) = 7$ but $f(-2) = 3$.

    No need to simplify as $(x-1)(x^2+x+1)/(x-1) = x^2 + x + 1$, but that also would be 'neither' from $f(2)$ and $f(-2)$.
13. Neither. It is not a function, because both $(2,-2)$ and $(2,2)$ are solutions, and the graph violates the vertical line test.
14. Both. There is only one point $(0,0)$. So the domain is $\{0\}$ (just $x = 0$). Conclude that both $f(-0) = f(0) = 0$ and $f(-0) = -f(0) = -0$ hold.
15. Even. It is a function because $f(x) = \sqrt{1-x^2}$ is a valid formula. Dispense with the $\pm$ of the solution once $y = f(x)$ is known to be non-negative. For each x in the domain, there will be exactly one y: the positive one. From that it is clear that the graph will pass the vertical line test. Had the y's not been stipulated to be positive, the equation would not be the equation of a function.

    If (a,b) is a solution, then $(-a,b)$ will also be a solution. That's all one needs to show the function is even.

    The procedure of separating a circle into two seimcircles is sometimes necessary to graph a circle on a graphing calculator. It is also used in higher mathematics when functions are needed and the relation at hand is the circle.