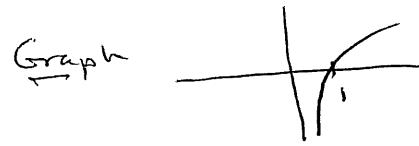


Essential Properties of Log and Exponential Functions

Def $\log_a x = y \leftrightarrow a^y = x$



Dom $x > 0$ Range $y \in \mathbb{R}$

Facts $\log x \equiv \log_{10} x$, $\ln x \equiv \log_e x$, $\log_a 1 = 0$ for all a ($a^0 = 1$)

$\log_b b = 1$ for any base.

Properties* $\log(xy) = \log x + \log y$, $\log\left(\frac{x}{y}\right) = \log x - \log y$

$\log(x^r) = r \log x$, $\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b} = \frac{\log_a x}{\log_a b}$

* If $\log x = \log y$ then $x = y$. Also, if $x = y$ then $\log x = \log y$.

Algorithm (example)
Suppose $a^x = b^{x+1}$. To solve for x , take the log of each side and apply properties and algebra to isolate x . It's best to choose base a or base b for best final form.

$$\begin{aligned} a^x = b^{x+1} : \log_a a^x &= \log_a b^{x+1} \\ x \log_a &= (x+1) \log_a b \\ x \cdot 1 &= x \log_a b + \log_a b \\ x - x \log_a b &= \log_a b \\ x(1 - \log_a b) &= \log_a b \\ x &= \frac{\log_a b}{1 - \log_a b} \end{aligned}$$

[or]

$$\begin{aligned} \log_b a^x &= \log_b b^{x+1} \\ x \log_b a &= (x+1) \log_b b \\ x \log_b a &= (x+1) \cdot 1 \\ x \log_b a - x &= 1 \\ x(\log_b a - 1) &= 1 \\ x &= \frac{1}{\log_b a - 1} \end{aligned}$$

This algorithm is used often to isolate (solve for) time in compound interest problems.

* For properties, assume true for any base.

Example

Suppose we seek how long it takes for an investment of \$1500 to reach \$2000 when it is invested at an annual rate of $1\frac{1}{2}\%$ and interest is compounded continuously. The set-up is:

$$F = Pe^{rt}, \quad F = 2000, P = 1500 \\ r = .015, t = ?$$

$$2000 = 1500 e^{.015t}$$

$$\textcircled{1} \quad \frac{2000}{1500} = e^{.015t} \quad (\text{divide by coeff})$$

$$\textcircled{2} \quad \frac{4}{3} = e^{.015t} \quad (\text{simplify, i.e., reduce})$$

$$\textcircled{3} \quad \ln\left(\frac{4}{3}\right) = \ln e^{.015t} \quad (\text{take ln of both sides})$$

$$\textcircled{4} \quad \ln\left(\frac{4}{3}\right) = .015t \ln e \quad (\text{property of logs}) \\ (\text{" " " " })$$

$$\textcircled{5} \quad \ln\left(\frac{4}{3}\right) = .015t \cdot 1$$

$$\textcircled{6} \quad \frac{\ln\left(\frac{4}{3}\right)}{.015} = t \quad (\text{algebra})$$