Solutions to page 1 of previous semester's exam:

1. Find the domain of each of the following functions. Express answers using interval notation.

a) $f(x)=\begin{cases}\frac{1}{x}, & \text{if } x \le -2\\ 3^x+4, & \text{if } x > -1\end{cases}$ Domain: $(-\infty,-2]\cup(-1,\infty)$

b) $g(x)=\dfrac{\sqrt{x+4}}{x^2-x-6}$, The domain relies only on the denominator. $x^2-x-6\neq 0$,

$(x-3)(x+2)=0$ at $x=3,-2$

Domain: $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$

$h(x)=\ln\left(x^2-9\right)$ The domain here relies on $x^2 - 9 > 0$, or $(x - 3)(x + 3) > 0$. Putting the roots on the number line and testing the intervals shows $x > 3$ or $x < -3$ satisfies the inequality. I don't much like this problem. But you should be able to find the domain for
$\ln(x + c)$ or $\ln(x - c)$, namely, $x > -c$ or $x > c$.

2. a) Factor the polynomial to find the roots:

$f(x) = 2x^4 - 6x^3 - 10x^2 = 2x^2(x^2 - 3x - 5) = 0$. So $x = 0$ or $x^2 - 3x - 5 = 0$. This doesn't factor, so use the quadratic formula, find the other two roots. $x=\dfrac{-3\pm\sqrt{29}}{2}$.

b) Solve the exponential equation to find the roots:

$g(x) = 3^{x-2} - 9 = 0$ $3^{x-2} = 9$ $3^{x-2} = 3^2$ $x - 2 = 2$ $x = 4$

3. a) Simply plug in $x = 5$ and evaluate the limit to get $125 - 1 + 1 - 32 = 93$.

b) You get the indeterminate form 0/0 when you plug in $x = 3$. Do algebra to reduce it.

Factoring gives $(2x + 1)(x - 3)$ in the numerator and $(x + 4)(x - 3)$ in the denominator. By cancellation, the expression becomes $(2x + 1)/(x + 4)$, which has a limit as $x \to 3$ of $7/7 = 1$.

c) Again you get the I.F. of 0/0 by plugging in $x = 2$. On this one it makes sense to rationalize the numerator and rationalize the denominator.

$$\lim_{x\to 2}\frac{\sqrt{3x+10}-4}{x^2-2x}=\lim_{x\to 2}\frac{\sqrt{3x+10}-4}{x(x-2)}\cdot\frac{\sqrt{3x+10}+4}{\sqrt{3x+10}+4}=\lim_{x\to 2}\frac{3x+10-16}{x(x-2)}=\lim_{x\to 2}\frac{3x-6}{x(x-2)}=\lim_{x\to 2}\frac{3(x-2)}{x(x-2)}=\lim_{x\to 2}\frac{3}{2}=\frac{3}{2}$$

d) At first I didn't like this problem, because you are expecting to get a form of infinity, as seen when you plug in $x = -4$ at the start. But by factoring you end up with a real number. But after all, any time you see a polynomial, go ahead and factor it if you can!

$$\lim_{x\to -4^-}\frac{x^2-x-20}{x+4}=\lim_{x\to -4^-}\frac{(x-5)(x+4)}{x+4}=\lim_{x\to -4^-}(x-5)=-4-5=-9$$

Suppose we had $\lim\limits_{x\to -4^-}\frac{-20}{x+4}$; then the answer would be positive infinity, since inspecting on the left of –4 gives tiny negative values in the denominator, say at x = –4.1, –4.01, and so the values are –20/–.1 = 200, –20/–.01 = 2000, trending to infinity.

(e) and (f) are good problems. There is a piecewise function where you evaluate the limits as x goes to the interval endpoint values, which is pretty easy. Then you note where it is or is not continuous, using the criteria that the value of x has to be in the domain, and the limit there (the RHL and the LHL) have to be equal. Watch:

$$f(x)=\begin{cases}3x+1, & \text{if } x<2\\ 5, & \text{if } x=2\\ \log_2(x+6)+4, & \text{if } x>2\end{cases}$$

Notice that the domain is all real numbers. Nothing is excluded. But the LHL = RHL = 7 as x goes to 2. But $f(2) = 5$.

LHL: $\lim\limits_{x\to 2^-} f(x)=7$ RHL: $\lim\limits_{x\to 2^+} f(x)=\log_2 8 + 4=3+4=7$, so $\lim\limits_{x\to 2} f(x)=7$

However, since $f(2)$ does not equal limit as x approaches 2, the function is discontinuous at $x = 2$.

4. a. Average velocity is simply the change in displacement over the change in time, that is,

$$\frac{s(6)-s(0)}{6-0}=\frac{37-1}{6}=\frac{36}{6}=6\text{ meters/min}$$

b.

$$s'(t) = \lim_{h\to 0} \frac{\left(3\left(t+h\right)^2 - 12(t+h) + 1\right) - \left(3t^2 - 12t + 1)\right)}{h}$$

$$= \lim_{h\to 0} \frac{3t^2 + 6th + 3h^2 - 12t - 12h + 1 - 3t^2 + 12t - 1}{h}$$

$$= \lim_{h\to 0} \frac{6th + 3h^2 - 12h}{h} = \lim_{h\to 0} 6t + 3h - 12 = 6t - 12$$

So $s'(t) = 6t - 12$

c. Instantaneous velocity at 5 minutes is found by evaluating $s'(5) = 30 - 12 = 18$ meters / min

d. $s(t) - s(1) = s'(1)(t - 1)$; $s'(1) = 6(1) - 12 = -6$ m/min

e. The object is not always moving forward. A negative velocity means it has changed direction and gone in the opposite direction of what is defined as forward.

I will add here that I didn't emphasize in my groups the displacement, velocity, acceleration aspect of derivatives. At least I didn't talk about it since the beginning. But I will be revisiting this soon when we do second order derivatives in the next unit.

5. My groups are skipping compound interest this exam. We revisit in final unit.

6. This is a linear cost revenue profit problem. You can expect one that includes a linear cost but a nonlinear revenue, since price *p* won't be fixed, so demand *q* will be a function of *p*.

I noticed that I didn't post the solutions to Sec 11. I won't be tonight, but the examples in the reading for out textbook are very clear.

If $p = 100 - 2q$, then $R(q) = pq = (100 - 2q)q = 100q - 2q^2$. So profit $P(q) = R(q) - C(q)$ as usual, finding a breakeven level of sales/production, finding marginals and interpreting what that means, and interpreting.

Topic 7. Expect you will have to graph and analyze a piecewise function for limits and continuity at relevant points.