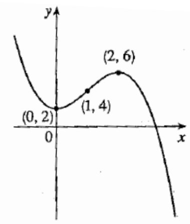
These are the problems you did on the curve sketching assignment from Dr Kazmierczak. In the solutions manual (oddly enough), the critical numbers are not explicitly named after the derivatives are shown, but you can see them from the intervals and extrema! Same for POI (or IP, for inflection points).

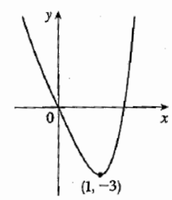
2. $y = f(x) = 2 + 3x^2 - x^3$ **A.** $D = \mathbb{R}$ **B.** y-intercept $= f(0) = 2$ **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 6x - 3x^2 = 3x(2 - x) > 0 \Leftrightarrow 0 < x < 2$, so f is increasing on $(0, 2)$ and decreasing on $(-\infty, 0)$ and $(2, \infty)$. **F.** Local maximum value $f(2) = 6$, local minimum value $f(0) = 2$ **G.** $f''(x) = 6 - 6x = 6(1 - x) > 0 \Leftrightarrow x < 1$, so f is CU on $(-\infty, 1)$ and CD on $(1, \infty)$. IP at $(1, 4)$

H.



3. $y = f(x) = x^4 - 4x = x(x^3 - 4)$ **A.** $D = \mathbb{R}$ **B.** x-intercepts are 0 and $\sqrt[3]{4}$, y-intercept $= f(0) = 0$ **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 4x^3 - 4 = 4(x^3 - 1) = 4(x - 1)(x^2 + x + 1) > 0 \Leftrightarrow x > 1$, so f is increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$. **F.** Local minimum value $f(1) = -3$, no local maximum **G.** $f''(x) = 12x^2 > 0$ for all x, so f is CU on $(-\infty, \infty)$. No IP

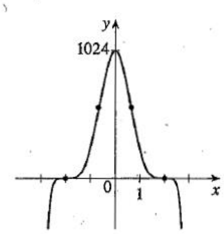
H.



8. $y = f(x) = (4 - x^2)^5$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 4^5 = 1024$; x-intercepts: ± 2 **C.** $f(-x) = f(x) \Rightarrow f$ is even; the curve is symmetric about the y-axis. **D.** No asymptote **E.** $f'(x) = 5(4 - x^2)^4(-2x) = -10x(4 - x^2)^4$, so for $x \neq \pm 2$ we have $f'(x) > 0 \Leftrightarrow x < 0$ and $f'(x) < 0 \Leftrightarrow x > 0$. Thus, f is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. **F.** Local maximum value $f(0) = 1024$

G.
$$f''(x) = -10x \cdot 4(4 - x^2)^3(-2x) + (4 - x^2)^4(-10)$$
$$= -10(4 - x^2)^3[-8x^2 + 4 - x^2] = -10(4 - x^2)^3(4 - 9x^2)$$
so $f''(x) = 0 \Leftrightarrow x = \pm 2, \pm\frac{2}{3}$. $f''(x) > 0 \Leftrightarrow -2 < x < -\frac{2}{3}$ and $\frac{2}{3} < x < 2$ and $f''(x) < 0 \Leftrightarrow x < -2, -\frac{2}{3} < x < \frac{2}{3}$, and $x > 2$, so f is CU on $(-\infty, 2)$, $\left(-\frac{2}{3}, \frac{2}{3}\right)$, and $(2, \infty)$, and CD on $\left(-2, -\frac{2}{3}\right)$ and $\left(\frac{2}{3}, 2\right)$. IP at $(\pm 2, 0)$ and $\left(\pm\frac{2}{3}, \left(\frac{32}{9}\right)^5\right) \approx (\pm 0.67, 568.25)$

H.



10. $y = f(x) = \dfrac{x^2 + 5x}{25 - x^2} = \dfrac{x(x + 5)}{(5 + x)(5 - x)} = \dfrac{x}{5 - x}$ for $x \neq -5$. There is a hole in the graph at $\left(-5, -\frac{1}{2}\right)$.

A. $D = \{x \mid x \neq \pm 5\} = (-\infty, -5) \cup (-5, 5) \cup (5, \infty)$ **B.** x-intercept $= 0$, y-intercept $= f(0) = 0$ **C.** No symmetry

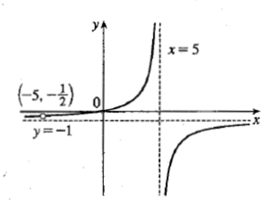
D. $\displaystyle\lim_{x \to \pm\infty} \frac{x}{5 - x} = -1$, so $y = -1$ is a HA. $\displaystyle\lim_{x \to 5^-} \frac{x}{5 - x} = \infty$, $\displaystyle\lim_{x \to 5^+} \frac{x}{5 - x} = -\infty$, so $x = 5$ is a VA.

E. $f'(x) = \dfrac{(5 - x)(1) - x(-1)}{(5 - x)^2} = \dfrac{5}{(5 - x)^2} > 0$ for all x in D, so f is increasing on $(-\infty, -5)$, $(-5, 5)$, and $(5, \infty)$. **F.** No extrema

G. $f'(x) = 5(5 - x)^{-2} \Rightarrow$

$f''(x) = -10(5 - x)^{-3}(-1) = \dfrac{10}{(5 - x)^3} > 0 \Leftrightarrow x < 5$, so f is CU on $(-\infty, -5)$ and $(-5, 5)$, and f is CD on $(5, \infty)$. No IP

H.



14. $y = f(x) = \dfrac{1}{x^2 - 4} = \dfrac{1}{(x+2)(x-2)}$ **A.** $D = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ **B.** No x-intercept, y-intercept $= f(0) = -\frac{1}{4}$ **C.** $f(-x) = f(x)$, so f is even; the graph is symmetric about the y-axis.

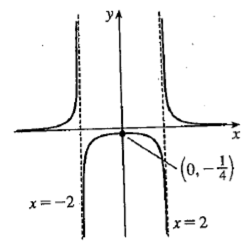
D. $\lim\limits_{x \to 2^+} \dfrac{1}{x^2 - 4} = \infty$, $\lim\limits_{x \to 2^-} f(x) = -\infty$, $\lim\limits_{x \to -2^+} f(x) = -\infty$, $\lim\limits_{x \to -2^-} f(x) = \infty$, so $x = \pm 2$ are VAs. $\lim\limits_{x \to \pm\infty} f(x) = 0$, so $y = 0$ is a HA. **E.** $f'(x) = -\dfrac{2x}{(x^2 - 4)^2}$ [Reciprocal Rule] > 0 if $x < 0$ and x is in D, so f is increasing on $(-\infty, -2)$ and $(-2, 0)$. f is decreasing on $(0, 2)$ and $(2, \infty)$.

F. Local maximum value $f(0) = -\frac{1}{4}$, no local minimum value

G. $$f''(x) = \frac{(x^2-4)^2(-2) - (-2x)2(x^2-4)(2x)}{[(x^2-4)^2]^2} = \frac{-2(x^2-4)\left[(x^2-4) - 4x^2\right]}{(x^2-4)^4} = \frac{-2(-3x^2-4)}{(x^2-4)^3} = \frac{2(3x^2+4)}{(x^2-4)^3}$$

$f''(x) < 0 \iff -2 < x < 2$, so f is CD on $(-2, 2)$ and CU on $(-\infty, -2)$ and $(2, \infty)$. No IP

H.



16. $y = f(x) = \dfrac{(x-1)^2}{x^2+1} \geq 0$ with equality $\iff x = 1$. **A.** $D = \mathbb{R}$ **B.** y-intercept $= f(0) = 1$; x-intercept 1 **C.** No symmetry **D.** $\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \dfrac{x^2 - 2x + 1}{x^2 + 1} = \lim\limits_{x \to \pm\infty} \dfrac{1 - 2/x + 1/x^2}{1 + 1/x^2} = 1$, so $y = 1$ is a HA. No VA

E. $f'(x) = \dfrac{(x^2+1)2(x-1) - (x-1)^2(2x)}{(x^2+1)^2} = \dfrac{2(x-1)\left[(x^2+1) - x(x-1)\right]}{(x^2+1)^2} = \dfrac{2(x-1)(x+1)}{(x^2+1)^2} < 0 \iff$

$-1 < x < 1$, so f is decreasing on $(-1, 1)$ and increasing on $(-\infty, -1)$ and $(1, \infty)$ **F.** Local maximum value $f(-1) = 2$, local minimum value $f(1) = 0$

G. $f''(x) = \dfrac{(x^2+1)^2(4x) - (2x^2-2)2(x^2+1)(2x)}{[(x^2+1)^2]^2} = \dfrac{4x(x^2+1)\left[(x^2+1) - (2x^2-2)\right]}{(x^2+1)^4} = \dfrac{4x(3-x^2)}{(x^2+1)^3}$.

$f''(x) > 0 \iff x < -\sqrt{3}$ or $0 < x < \sqrt{3}$, so f is CU on $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$, and f is CD on $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$.

$f(\pm\sqrt{3}) = \frac{1}{4}(\sqrt{3} \mp 1)^2 = \frac{1}{4}(4 \mp 2\sqrt{3}) = 1 \mp \frac{1}{2}\sqrt{3}$ [$\approx 0.13, 1.87$], so there are IPs at $(-\sqrt{3}, 1 + \frac{1}{2}\sqrt{3})$, $(0, 1)$, and $(\sqrt{3}, 1 - \frac{1}{2}\sqrt{3})$. Note that the graph is symmetric about the point $(0, 1)$.

H.