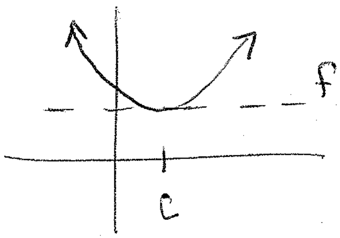
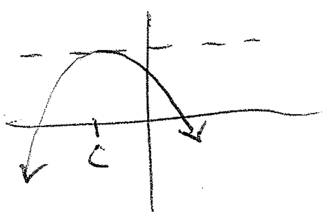
Critical points (that is, numbers)
An overview

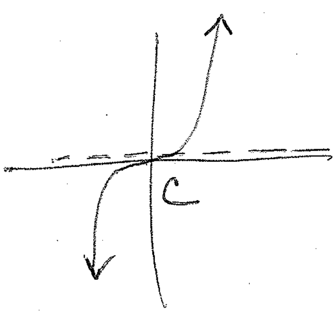
All the c in these graphs represent critical points of f.



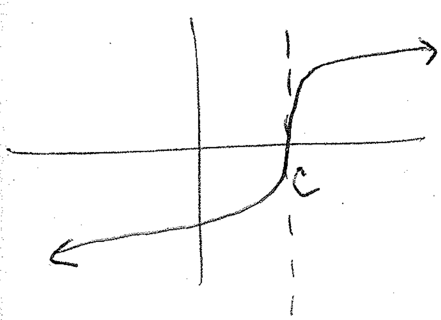
$f'(c)=0$, $f(c)$ is a local min



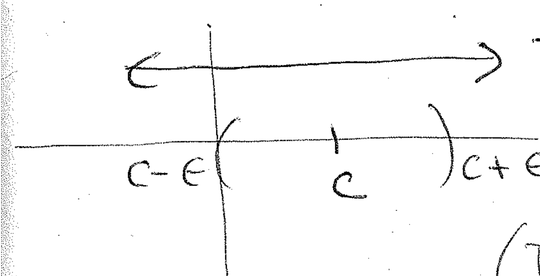
$f'(c)=0$, $f(c)$ is a local max



$f'(c)=0$, $f(c)$ is neither a local max nor min

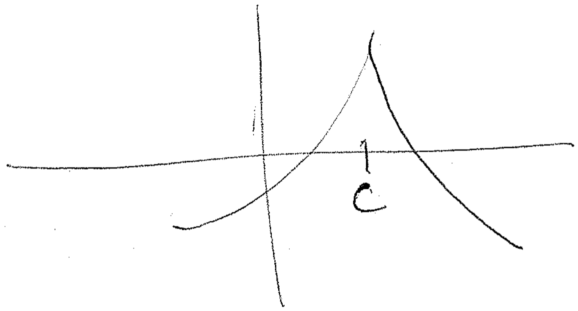


$f'(c)$ DNE



$f'(c)=0$, $f(c)$ is both a local max <u>and</u> a local min b/c $f(x)=f(c)$ in $(c-\epsilon, c+\epsilon)$

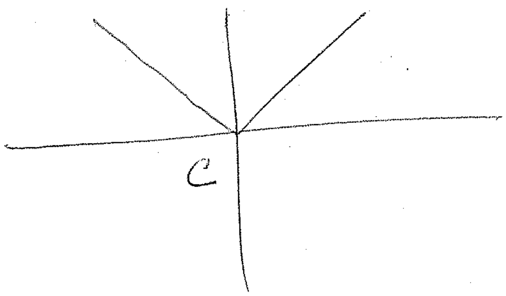
(By definition, for all x in an open n'hood of c, $f(x) \le f(c)$ implies $f(c)$ is a local max, and $f(x) \geq f(c)$ implies $f(c)$ a local min)



c is a cusp

$f'(c)$ DNE

$f(c)$ is a local max



c is a corner

$f'(c)$ DNE

$f(c)$ is a local min

A note about abs value fcn $f(x) = |x|$

We claim f is not differentiable at $c = 0$.

That ~~in~~ is, $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ DNE.

To prove this, we write the fcn in its correct piecewise fcn. form and consider the difference quotients of each piece.

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

for $x \geq 0$
$f(x) = x$

$$\lim_{h \to 0} \frac{(x+h) - x}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

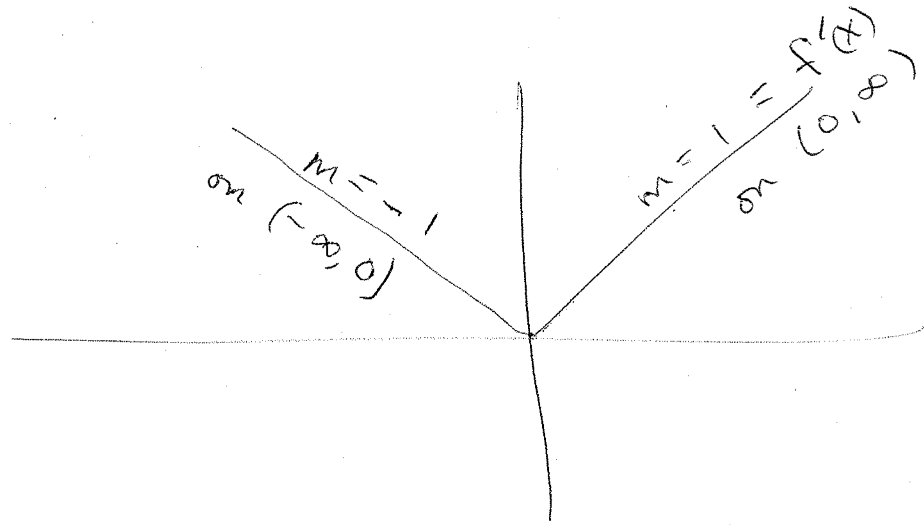
for $x<0$
$f(x) = -x$

$$\lim_{h \to 0} \frac{-(x+h) - (-x)}{h} = \lim_{h \to 0} \frac{-h}{h}$$

$$= -1$$

So we see that $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ DNE

Since the LHL, effectively, the slope of $f(x) = |x|$ on $(-\infty, 0)$, does not equal the slope of $f(x) = |x|$ on $[0, \infty)$, that is, the RHL.



The slope jumps from -1 to 1 at $x = 0$, therefore, f is not differentiable at $x = 0$, which is still a critical pt and $f(0)$ a local min.