Compounding Interest

P = present value or principal
F = future value
n = number of compoundings / year
r = interest rate (here $r = 3\%$)

$n=1$ $P + P\left(\frac{.03}{12}\right) = P\left(1 + \frac{.03}{12}\right)$

$n=2$ Take int on $P\left(1+\frac{.03}{12}\right)$ + add to $P\left(1+\frac{.03}{12}\right)$

$n=12$ So $P\left(1+\frac{.03}{12}\right) + P\left(1+\frac{.03}{12}\right)\left(\frac{.03}{12}\right)$

factor out $P\left(1+\frac{.03}{12}\right)$

$$P\left(1+\frac{.03}{12}\right)\left[1 + \frac{.03}{12}\right]$$

$$= P\left(1+\frac{.03}{12}\right)^2 \quad \text{at } n=2$$

n can go to more than 12.
Say $n = 21$.

Then $P\left(1+\frac{.03}{12}\right)^{21}$ $21 = nt$ where n = times/yr + t = yr

$12 \frac{\text{times}}{\text{yr}} \cdot 21 \cancel{\text{mos}} \cdot \frac{1 \text{ yr}}{12 \cancel{\text{mos}}}$

After 1 year, if $n = 52$,

then $P\left(1+\frac{.03}{52}\right)^{52}$ is value

vs $P\left(1+\frac{.03}{12}\right)^{12}$ for monthly

Sec 5 Important ideas:

future value $F = P\left(1+\frac{r}{n}\right)^{nt}$

present value $P = F\left(1+\frac{r}{n}\right)^{-nt}$

n = discrete number of compounding

Look at table on p 51. The effective int rate increases with increasing n. It has a limit, which is determined as follows.

For the sake of simplicity, suppose $r = 100\%$. That is, the return on your investment is double your principal.

Then $\boxed{F = P\left(1+\frac{1}{n}\right)^{nt}}$

Suppose further that $t = 1$ year.

Then $F = P\left(1+\frac{1}{n}\right)^{n}$

As n increases, the table on p. 52 shows what the principal's multiplier does.

We say $\boxed{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = 2.718\ldots = e}$

Of course r won't be 100%. For any fixed r, then, the multiplier is determined by this: Let $m = n/r$

$$\left(1+\frac{r}{n}\right)^{n} = \left(\left(1+\frac{r}{n}\right)^{\frac{n}{r}}\right)^{r} = \left(\left(1+\frac{1}{m}\right)^{m}\right)^{r}$$

which again, as n gets large

Thus $\left[\left(1+\frac{1}{m}\right)^m\right]^r \xrightarrow[n\to\infty]{} e^r$.

$m = \frac{n}{r} \to \infty$

Finally, $F = Pe^{rt}$ as time passes

$\boxed{\text{Effective interest rate} = e^r - 1}$

The book's description on p. 51 is incorrect. It says eff int rate is "rate if int were compounded annually". Clearly, this would be simple interest. It should read "... if interest were paid at 1 year maturity" (APY in bank lingo)

Ex 5.6 Error: $F = P\left(1+\frac{.105}{1}\right)^{(1)(2)} \leftarrow nt$,

$t = 2$

$n = 1$

Exponential fn: $f(x) = e^x$ or $\exp(x)$