FUNCTION COMPOSITION

$$(f \circ g)(x) = f(g(x))$$

To calculate a composition of function we will evaluate the inner function and substitute the answer into the outer function. This is shown in the following example.

Example 521.

$$\begin{aligned} a(x) &= x^2 - 2x + 1 \\ b(x) &= x - 5 && \text{Rewrite as } a \text{ function in function} \\ \text{Find } &(a \circ b)(3) \end{aligned}$$

$$\begin{aligned} a(b(3)) & && \text{Evaluate the inner function first, } b(3) \\ b(3) &= (3) - 5 = -2 && \text{This solution is put into } a,\ a(-2) \\ a(-2) &= (-2)^2 - 2(-2) + 1 && \text{Evaluate} \\ a(-2) &= 4 + 4 + 1 && \text{Add} \\ a(-2) &= 9 && \text{Our Solution} \end{aligned}$$

We can also evaluate a composition of functions at a variable. In these problems we will take the inside function and substitute into the outside function.

Example 522.

$$\begin{aligned} & f(x) = x^2 - x \\ & g(x) = x + 3 && \text{Rewrite as } a \text{ function in function} \\ & \text{Find } (f \circ g)(x) \end{aligned}$$

$$\begin{aligned} f(g(x)) & && \text{Replace } g(x) \text{ with } x + 3 \\ f(x + 3) & && \text{Replace the variables in } f \text{ with } (x + 3) \\ (x + 3)^2 - (x + 3) & && \text{Evaluate exponent} \\ (x^2 + 6x + 9) - (x + 3) & && \text{Distribute negative} \\ x^2 + 6x + 9 - x - 3 & && \text{Combine like terms} \\ x^2 + 5x + 6 & && \text{Our Solution} \end{aligned}$$

It is important to note that very rarely is $(f \circ g)(x)$ the same as $(g \circ f)(x)$ as the following example will show, using the same equations, but compositing them in the opposite direction.

Example 523.

$$f(x) = x^2 - x$$
$$g(x) = x + 3 \quad \text{Rewrite as } a \text{ function in function}$$
$$\text{Find } (g \circ f)(x)$$

$g(f(x))$	Replace $f(x)$ with $x^2 - x$
$g(x^2 - x)$	Replace the variable in g with $(x^2 - x)$
$(x^2 - x) + 3$	Here the parenthesis don't change the expression
$x^2 - x + 3$	Our Solution

World View Note: The term "function" came from Gottfried Wihelm Leibniz, a German mathematician from the late 17th century.

41) $f(x) = -4x + 1$
$g(x) = 4x + 3$
Find $(f \circ g)(9)$

42) $g(x) = x - 1$
Find $(g \circ g)(7)$

43) $h(a) = 3a + 3$
$g(a) = a + 1$
Find $(h \circ g)(5)$

44) $g(t) = t + 3$
$h(t) = 2t - 5$
Find $(g \circ h)(3)$

45) $g(x) = x + 4$
$h(x) = x^2 - 1$
Find $(g \circ h)(10)$

46) $f(a) = 2a - 4$
$g(a) = a^2 + 2a$
Find $(f \circ g)(-4)$

47) $f(n) = -4n + 2$
$g(n) = n + 4$
Find $(f \circ g)(9)$

48) $g(x) = 3x + 4$
$h(x) = x^3 + 3x$
Find $(g \circ h)(3)$

49) $g(x) = 2x - 4$
$h(x) = 2x^3 + 4x^2$
Find $(g \circ h)(3)$

50) $g(a) = a^2 + 3$
Find $(g \circ g)(-3)$

51) $g(x) = x^2 - 5x$
$h(x) = 4x + 4$
Find $(g \circ h)(x)$

52) $g(a) = 2a + 4$
$h(a) = -4a + 5$
Find $(g \circ h)(a)$

53) $f(a) = -2a + 2$
$g(a) = 4a$
Find $(f \circ g)(a)$

54) $g(t) = -t - 4$
Find $(g \circ g)(t)$

55) $g(x) = 4x + 4$
$f(x) = x^3 - 1$
Find $(g \circ f)(x)$

56) $f(n) = -2n^2 - 4n$
$g(n) = n + 2$
Find $(f \circ g)(n)$

57) $g(x) = -x + 5$
$f(x) = 2x - 3$
Find $(g \circ f)(x)$

58) $g(t) = t^3 - t$
$f(t) = 3t - 4$
Find $(g \circ f)(t)$

59) $f(t) = 4t + 3$
$g(t) = -4t - 2$
Find $(f \circ g)(t)$

60) $f(x) = 3x - 4$
$g(x) = x^3 + 2x^2$
Find $(f \circ g)(x)$