Nov 18 Final domain technique - first we need solving quadratic inequalities

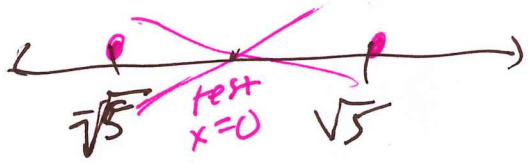
Ex $x^2 - 5 \geq 0$ Find all x that satisfy this quadratic inequality! Answer will be intervals.

Step 1 Find the roots of corresponding equation

$x^2 - 5 = 0$
$x^2 = 5$
$\sqrt{x^2} = \pm\sqrt{5}$
$x = \pm\sqrt{5}$

Step 2 Plot the roots on a number line; use dots if inequality given is $\geq, \leq 0$; use holes if $>, <$

Step 3 Pick an "easy pt" (whole #) in each interval and see if it satisfies the original inequality



$x^2 - 5 \geq 0$
$0^2 - 5 \geq 0$
$0 - 5 \geq 0$
$-5 \geq 0$ not true

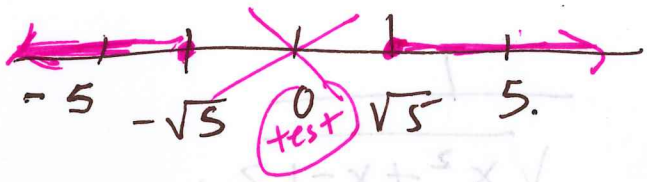
If test pt. does not satisfy the inequality, then discard the (whole) interval where test pt.

FACT $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$

"more"

What whole # is closest to $\sqrt{5}$ on right?

$$2 < \sqrt{5} < 3$$
$$2^2 \quad \sqrt{5}^2 \quad 3^2$$
$$4 \quad 5 \quad 9$$



$x^2 - 5 \geq 0$

$(-5)^2 - 5 = 25 - 5 = 20 \geq 0$ good, checks

$5^2 - 5 = 25 - 5 = 20 \geq 0$ " "

Step 4 write the intervals of soln:

Rf. $\boxed{(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty) \quad \text{soln set}}$

We could have picked any # in the intervals we formed, but we need only one value per interval.

Yesterday $f(x) = \dfrac{1}{x^2+x-12}$

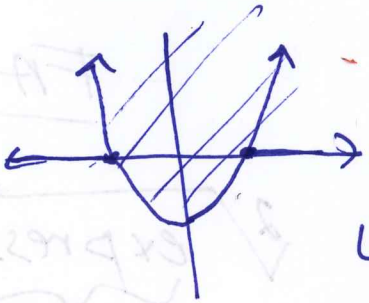
$D_f: (-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

Today $f(x) = \dfrac{1}{\sqrt{x^2+x-12}}$

$D_f: (-\infty, -4) \cup (3, \infty)$

Ex $f(x) = x^2 - 1 \geq 0$

solid curve



Later

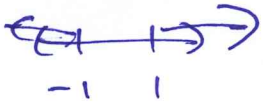
$y \geq 0$

Ex Domain $f(x) = \sqrt{x^2-1}$ Need radicand to be nonneg.

$x^2 - 1 \geq 0$

$x^2 \geq 1$ $x = \pm 1$

x ?



D_f: $(-\infty, -1] \cup [1, \infty)$

Ex $g(x) = \dfrac{1}{\sqrt{x^2-1}}$ D_g: $(-\infty, -1) \cup (1, \infty)$

Ex $f(x) = \dfrac{x^2 + 3x - 4}{\sqrt{x^2-4}}$ Same approach; top doesn't matter since it's simple poly.

FACT

$D_{poly} = \mathbb{R}$

no $\sqrt{\ }$, no fraction

$x^2 - 4 > 0$

$x = \pm 2$

D_f:

$(-\infty, -2) \cup (2, \infty)$

FACT

$\sqrt[2]{\underbrace{\text{expression}}_{\geq 0}}$

$\sqrt[3]{\underbrace{\text{expression}}_{\mathbb{R}}}$

even roots are restricted to non-negative

odd roots have no restrictions

$\sqrt[4]{}$

$\sqrt[5]{}$

$\sqrt[6]{}$

$\sqrt[7]{}$

$\vdots$

$\vdots$

Wksht #14

1. $f(x) = x^2 - 1$, $f(-\sqrt{3})$

$f(-\sqrt{3}) = (-\sqrt{3})^2 - 1 = 3 - 1 = 2$

$f(\sqrt{7}) = (\sqrt{7})^2 - 1 = 7 - 1 = 6$

$f(0) = 0^2 - 1 = 0 - 1 = -1$

2. $f(x) = -2x^2 + x$

$f(-5) = -2(-5)^2 + -5 = -50 - 5$

$-2(25) - 5 = -55$

$f(-\frac{1}{2}) = ?$

$f(-\frac{1}{2}) = -\frac{2}{1}(-\frac{1}{2})^2 + (-\frac{1}{2})$

$= (-2)(-\frac{1}{2})(-\frac{1}{2}) + -\frac{1}{2}$

PEMDAS

$= (\frac{-2}{1})(\frac{1}{4}) - \frac{1}{2} = -\frac{1}{2} - \frac{1}{2} = -1$

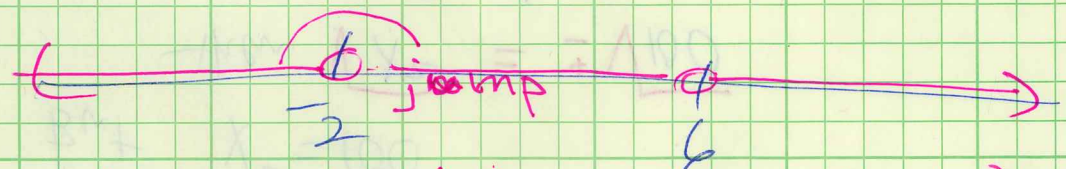
18. $f(x) = \dfrac{x+1}{x^2 - 4x - 12}$ → Good!

$x^2 - 4x - 12 \neq 0$

$(x - 6)(x + 2) \neq 0$

$x - 6 \neq 0 \rightarrow x \neq 6$

$x + 2 \neq 0 \rightarrow x \neq -2$



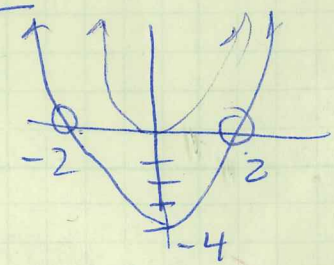
$(-\infty, -2) \cup (-2, 6) \cup (6, \infty)$

Review Alone $\sqrt{4} =$ means 2

But $x^2 = 4 \rightarrow \sqrt{x^2} = \pm\sqrt{4}$

then $x = 2, -2$

$y = x^2 - 4$



So $\sqrt{100} = 10$ only

But $x^2 = 100$

then $\sqrt{x^2} = \pm\sqrt{100}$

$x = \pm 10$

Reviewed WH 227 Monday after T-giving 1:30-3:30

Nov 28

Write it:

11-20 Domain - set of x for which y is defined (y exists)

11. $f(x) = \sqrt{4x-2}$ Set $4x-2 \geq 0$

$x \geq 1/2$ $[1/2, \infty)$

Domain: summary of restrictions

$\frac{\#}{\#}$ ← expression here $\neq 0$

$\sqrt{\#}$ ← expression cannot be negative

Cryptic puzzle

$\frac{\#}{\sqrt{\text{expression}}}$ expression > 0

$f(x) = \frac{\text{top}}{\text{expression}}$ → if no $\sqrt{\ }$, no restriction expression $\neq 0$

$f(x) = \sqrt{\text{expression}}$ expression ≥ 0

$f(x) = \frac{\text{top}}{\sqrt{\text{expression}}}$ expression > 0

$f(x) = \frac{\sqrt{\boxed{\text{expression 1}}}}{\boxed{\text{expression 2}}}$ $\left.\begin{matrix} \geq 0 \\ \neq 0 \end{matrix}\right\}$ intersection of these sets

For example:

$f(x) = \frac{x^2 + 3x - 7}{x}$ → fine; x only trouble

D_f: $x \neq 0$ $(-\infty, 0) \cup (0, \infty)$ *

$f(x) = \sqrt{18 - 3x}$ → cannot be negative

D_f: $18 - 3x \geq 0$

$\frac{-3x}{-3} \geq \frac{-18}{-3}$

$x \leq 6$

$(-\infty, 6]$ *

$f(x) = \frac{\sqrt{x}}{x^2 - 4}$ $\sqrt{x}$ non-negative; $x^2 - 4$ none-zero

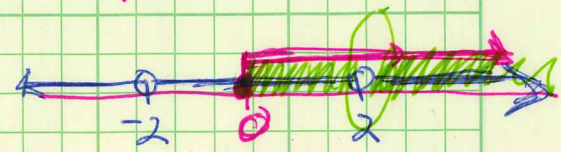
$x \geq 0$ and $x^2 - 4 \neq 0$

$\cap$

$x \neq \pm 2$

from $\sqrt{x^2} = \sqrt{4}$

$x = \pm\sqrt{4}$



$[0, 2) \cup (2, \infty)$

jump over 2

#20 $f(x) = \dfrac{x^2-9}{x^2-2x-1}$

D_f: ? $\underbrace{x^2-2x-1 \neq 0}_{\text{roots} = ?}$

ax^2+bx+c

QF $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{2 \pm \sqrt{4-4(1)(-1)}}{2(1)}$ $\sqrt{8}$

$x = \dfrac{\cancel{2} \pm \cancel{2}\sqrt{2}}{\cancel{2}}$

$= 1 \pm \sqrt{2}$

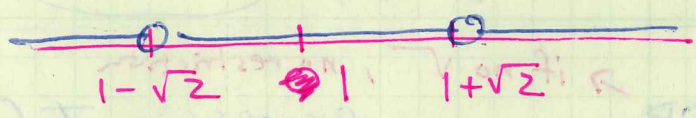
$\sqrt{2} \approx 1.414$

$\sqrt{8} = \sqrt{4 \cdot 2}$

$= \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

D_f:

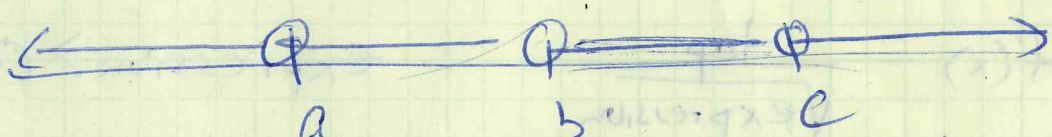


$(-\infty, 1-\sqrt{2})$

$\cup (1-\sqrt{2}, 1+\sqrt{2})$

$\cup (1+\sqrt{2}, \infty)$

$\sqrt{1 \cdot 1} = \sqrt{1}\ \sqrt{2}\ \sqrt{2 \cdot 2} = \sqrt{4}$



$(-\infty, a) \cup (a, b) \cup (b, c) \cup (c, \infty)$

11/21/22

- Finding y-intercept, roots + vertex of parabola (quadratic fcn) using both forms
  $y = ax^2 + bx + c$ & $y - k = \#(x-h)^2$

- Graph it

Ex $f(x) = 1x^2 - 4x - 5$

$f(0) = 0^2 - 4(0) - 5$

$= -5$

y-int: $f(0) = -5$ $(0,-5)$

roots: $x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = 5, -1$

$(5,0), (-1,0)$

Vertex by way of

$f(x) = ax^2 + bx + c$

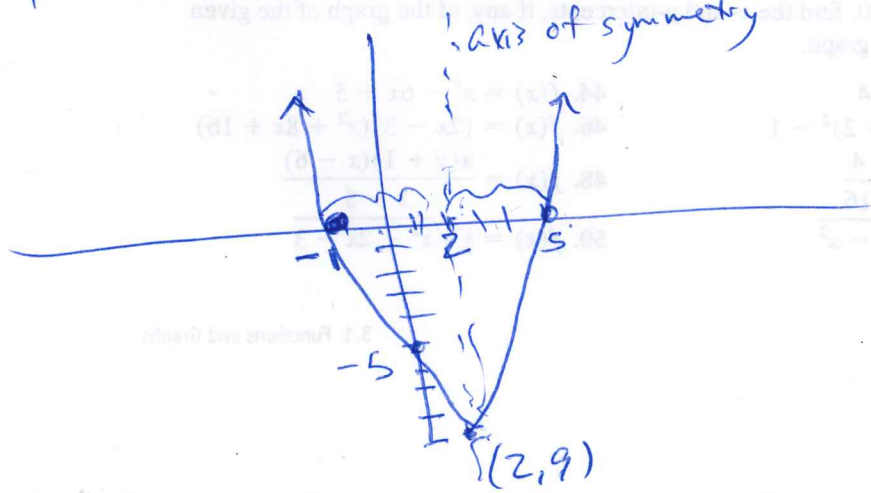
$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$\left(\frac{-4}{2(1)} = 2, f(2) = -9\right)$ vertex $(2,-9)$

Wonderful way to get features if a, b, c are small, even, $a = 1$

So alternate vertex way is helpful (video)

First, let's graph this:



Vertex (h,k) form

Same example, put in $y-k=(x-h)^2$ "(h,k) form"

$f(x)=x^2-4x-5$. given

$y=x^2-4x-5$. rewrite as y for f(x)

$y+5=x^2-4x$. move constant to left

$4+y+5=x^2-4x+\left(\frac{-4}{2}\right)^2$. complete the square

$y+9=x^2-4x+4$. identify the completed sq.

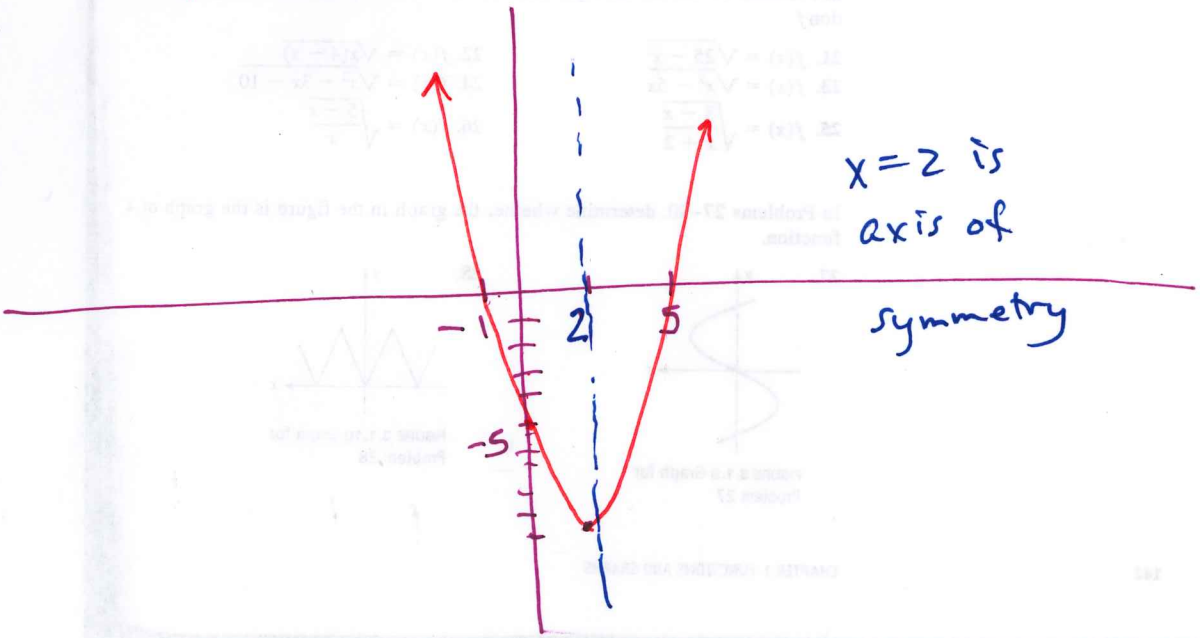
$y+9=(x-2)^2$ from $\left(x-\frac{b}{2}\right)^2$

$y-k=(x-h)^2$. match up constants with h,k

$h=2$

$k=-9$

Knew $(0,-5)$, $(-1,0)$, $(5,0)$



x=2 is axis of symmetry

Ex $y = 2x^2 - 12x + 10$ $\longrightarrow$ $2(x^2 - 6x + 5)$ $(0, 10)$

$2(x-5)(x-1) = 0$

$x = 5, 1$

$\cancel{y = 2x^2}$

$18 + y - 10 = 2(x^2 - 6x + 9)$

$\frac{-6}{2}$

$y + 8 = 2(x - 3)^2$

$y - -8$

$y - k$ $\quad x - h$

$\left(\frac{b}{2}\right)^2$

$(3, -8)$ vertex

Domain Ex $f(x) = \dfrac{x-7}{\sqrt{x^2-3x-4}}$

D_f • Note the restrictions - top - none

bottom - $x^2-3x-4>0$

$\neq 0$ b/c in deno

$\not> 0$ b/c under $\sqrt{\ }$

treat like eqn to get roots

• $x^2-3x-4=0$

Find the roots

$(x-4)(x+1)=0$

$x=4,-1$

• $x=-2$ $\boxed{-1}$ $x=0$ $\boxed{4}$ $x=5$

~~Put~~ Mark roots on # line with ○ or ●

$\sqrt{\ }$ in bottom $\sqrt{\ }$ on top

• Test pt in each interval formed into the inequality

$(x-4)(x+1)>0$

$0-4$ $0+1$

$\ominus$ $\oplus$

$\ominus$ $\not> 0$

$5-4$ $5+1>0$

$\oplus$ $\oplus$

$\cancel{5t-2}$ $\cancel{5+2}$

$\oplus$

$(-2-4)$ $(-2+1)>0$

$\ominus$ $\ominus$

• $(-\infty,-1)\cup(4,\infty)$

Quiz #24

$f(x)=\dfrac{x+9}{\sqrt{x^2+x-72}}$

CASE 3: FRACTION & RADICAL COMBINATION

Directions: State the domain in interval notation.

13.) $f(x) = \frac{5}{\sqrt{2x-10}}$

$2x-10>0$

$x>5$

$(5,\infty)$

14.) $f(x) = \frac{9}{\sqrt{x^2-144}}$

$x^2-144>0$

$x=\pm 12$ root

$-12 \quad x=0 \quad 12$

$x^2-144=0^2-144<0$

$(-\infty,-12)\cup(12,\infty)$

15.) $f(x) = \frac{11}{\sqrt{x^2-100}}$

$(-\infty,-10)\cup(10,\infty)$

16.) $y = \frac{\sqrt{x^2-36}}{2x-8}$

$x\neq 4 \cap (-\infty,-6]\cup[6,$

$-6 \quad 4 \quad 6$

17.) $y = \frac{\sqrt{x^2-25}}{3x-24}$

$x\neq 8 \cap (-\infty,-5]\cup[5,\infty)$

$-5 \quad 5 \quad 8$

$(-\infty,-5]$
$\cup[5,8)\cup(8,\infty)$

18.) $y = \frac{\sqrt{9-x^2}}{x^2+7x+10}$

$9-x^2\geq 0$

$[-3,3]$

$\cap$

$x^2+7x+10$

$x\neq -2,-5$

$-5 \quad -3 \quad -2 \quad 3$

$[-3,2)\cup(2,$

MIXED PRACTICE: State the domain in interval notation.

19.) $y = \frac{5x^3-9}{x^3+13x^2+42x}$ → any $\mathbb{R}$ → factor!

$x(x^2+13x+42)\neq 0$

$=x(x+7)(x+6)\neq 0$

$x\neq 0,-7,-6$

$(-\infty,-7)\cup(-7,-6)$
$\cup(-6,0)\cup(0,\infty)$

20.) $f(x) = \frac{\sqrt{x^2-9x-18}}{x^2-16+63}$

21.) $f(x) = \frac{\sqrt{x^2-7x-18}}{x^2-5x-14}$

$x^2-7x-18\geq 0 \cap x^2-5x-14\neq 0$

$(x-9)(x+2)\geq 0 \cap (x-7)(x+2)\neq 0$

$-2 \quad 7 \quad 9$

$x\neq$

$(-\infty,-2)\cup[9,\infty)$

22.) $f(x) = \sqrt{25-5x}$

23.) $f(x) = 8x^3-13x^2+9x-4$

x can be any $\mathbb{R}$

$(-\infty,\infty)$ – Polynomial domain $=\mathbb{R}$

24.) $f(x) = \frac{x+9}{\sqrt{x^2+x-72}}$ – Quiz question

$x+9$: any $x\in\mathbb{R}$ $(-\infty,\infty$

$x^2+x-72>0$; $(x-8)(x+9)$

$-9 \quad +8$

$(-\infty,-9)\cup$
$(8,\infty$

25.) $f(x) = \frac{\sqrt{7x^2-31x-20}}{7x^2+9x}$

$(-\infty,0]$
$\cup[6,\infty)$

26.) $f(x) = \sqrt{8x^2-48x}$

$8x^2-48x$

$=8x(x-6)\geq 0$

$+ \quad - \quad +$

$x=-1 \quad 0 \quad x=1 \quad 6 \quad x=7$

Pre-Calculus/Trig
UNIT 1: Algebra II Review – SECTION 3 WORKSHEET

Name: Key
Date:

FINDING DOMAIN ALGEBRAICALLY

CASE 1: FRACTIONS

Directions: State the domain in interval notation.

1.) $y = \frac{1}{x}$

D_y: $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

2.) $y = \frac{1}{x-2}$ $x \neq 2$

D_y: $(-\infty, 2) \cup (2, \infty)$

3.) $f(x) = \frac{8}{3x+9}$ $x \neq -3$

D_f: $(-\infty, -3) \cup (-3, \infty)$

4.) $f(x) = \frac{x^3}{x^2-4x-96}$ → Top – no issues

→ $\neq 0$

$(x-12)(x+8) \neq 0$

$=0$ $=0$

$x \neq 12, -8$

$(-\infty, -8) \cup (-8, 12) \cup (12, \infty)$

5.) $y = \frac{6x}{x^2+7x+12}$

$x^2 + 7x + 12 \neq 0$

$(x+4)(x+3) \neq 0$

$x \neq -4, -3$

6.) $y = \frac{3x^2-8x}{2x^2-5x-3}$

$2x^2 - 5x - 3 \neq 0$

$(2x+1)(x-3) = 0$

$x \neq -\frac{1}{2}, 3$

CASE 2: RADICALS

Directions: State the domain in interval notation.

7.) $y = \sqrt{x-3}$ $x - 3 \geq 0$

$x \geq 3$ $[3, \infty)$

8.) $y = \sqrt{2x+8}$ $2x + 8 \geq 0$

$x \geq -4$

$[-4, \infty)$

So must any even root radicand

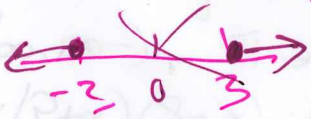
Square root radicand must be ≥ 0;

Cube root radicands can be all $\mathbb{R}$.

So can any odd root radicands

9.) $y = \sqrt{x^2-9}$ $x^2 - 9 \geq 0$

$(x-3)(x+3) \geq 0$



$0-3$ $0+3$

$\ominus$ $\oplus$ < 0

10.) $y = \sqrt[3]{1-x^2}$

$(-\infty, \infty)$

11.) $f(x) = \sqrt{4-x^2}$

$4 - x^2 \geq 0$, $x = \pm 2$

D_f: $[-2, 2]$

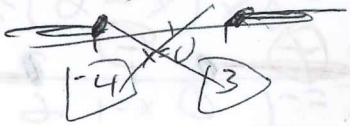
12.) $f(x) = \sqrt{x^2+x-12}$

$(-\infty, -4] \cup [3, \infty)$

from: $x^2 + x - 12 \geq 0$

$(x-3)(x+4) \geq 0$

test $x = 0$ ← into fact



$(0-3)(0+4)$

$\ominus$ $\oplus$ < 0