

Nov 11

Veteran's Day



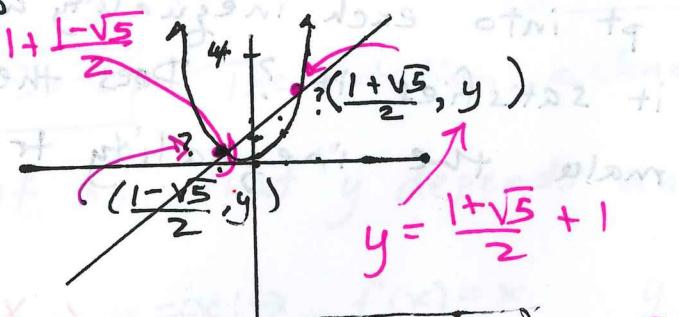
Graphing Inequalities

Systems of inequalities!

$$\begin{cases} y = x + 1 \\ y = x^2 \end{cases}$$

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0$$



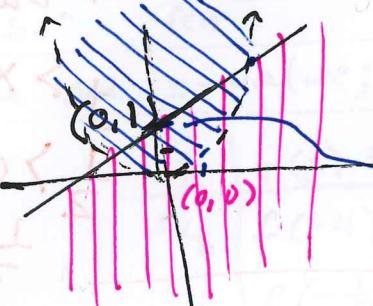
- Graph like eqns

- Use --- or —
as needed

- Shade region
of solution set

- Check point in
or outside region

$$\begin{cases} y \leq x + 1 \\ y \leq 0 + 1 = 1 \\ y > x^2 \end{cases}$$



(0, 0) is a great test point!

"Solution set"

$$\begin{aligned} y &> x^2 & y &\leq x+1 \\ \frac{1}{2} &> 0^2 = 0 & \frac{1}{2} &\leq 0+1 \\ \frac{1}{2} &> 0 & \frac{1}{2} &\leq 1 \end{aligned}$$

$$\begin{cases} y < 3 \\ y > x^2 - 1 \\ x^2 - 1 = 0 \\ x = \pm 1 \end{cases}$$



$$\begin{cases} y > 0 \\ y < 2x + 5 \end{cases}$$

When we graph
two, my vertical
line drawn through
the curve will
intersect it.

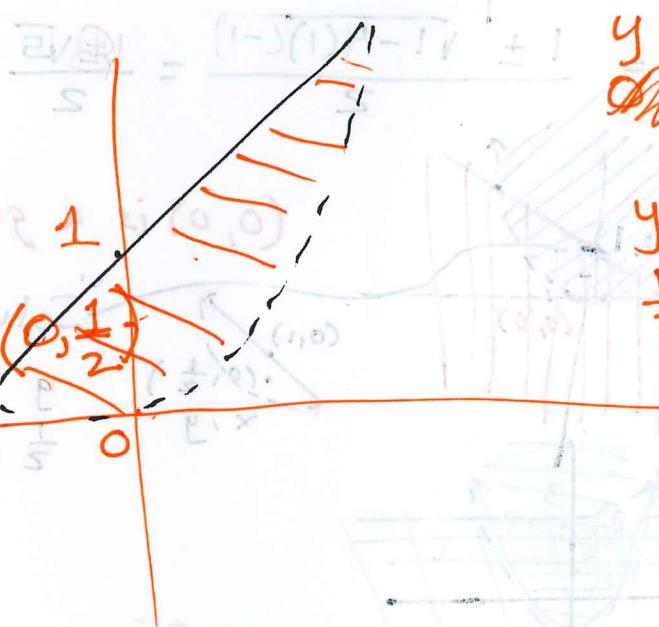
due to the uniqueness
of y

What does checking a point look like? 11 vol

Pick pt. clearly in or outside the shaded region.

(x, y)

Plug pt into each inequality and see if it satisfies it? Does the (x, y) chosen make the inequality true?



$$y \leq x + 1$$

$$y > x^2$$

$$\frac{1}{2} > 0^2$$

$$\frac{1}{2} > 0$$

$(0, \frac{1}{2})$

$$\frac{1}{2} \leq 0 + 1 = 1$$

true

$$\frac{1}{2} > 0$$

true

$$x^2 \geq 0$$

$$x^2 \leq 0$$

$$x^2 = 0$$

$$x^2 < 0$$

$$x^2 > 0$$

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Veteran's Day - Functions

"y is a function of x" \equiv "y depends uniquely on x"

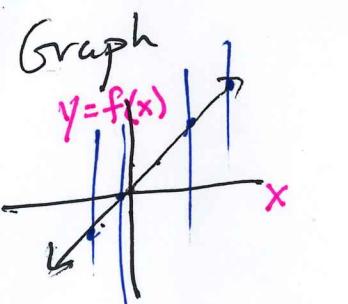
Given function $y = f(x)$, where $y = 3x - 5$, we now write $f(x) = 3x - 5$. The notation acknowledges that

"y is a fn of x," that "y depends uniquely on x."

Three fns: $y = x \rightarrow f(x) = x$ $y = x^2 \rightarrow f(x) = x^2$

"humble fcn" *Identity fcn* $y = \sqrt{x} \rightarrow f(x) = \sqrt{x}$

"you give me an x-value; I'll give you the y-value."

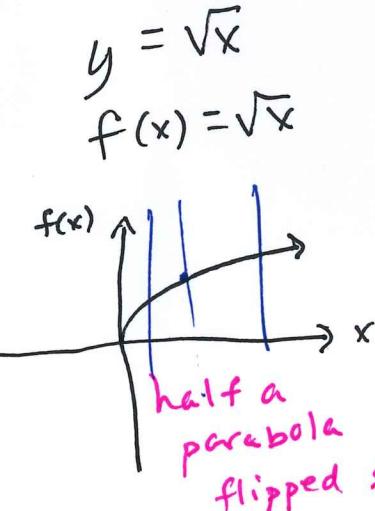


x	$f(x) = x$
-4	$f(-4) = -4$
$-\frac{1}{2}$	$f(-\frac{1}{2}) = -\frac{1}{2}$
0	$f(0) = 0$
1	1
2	2
$\sqrt{5}$	$\sqrt{5}$
11,432	11,432

x is y

y is x

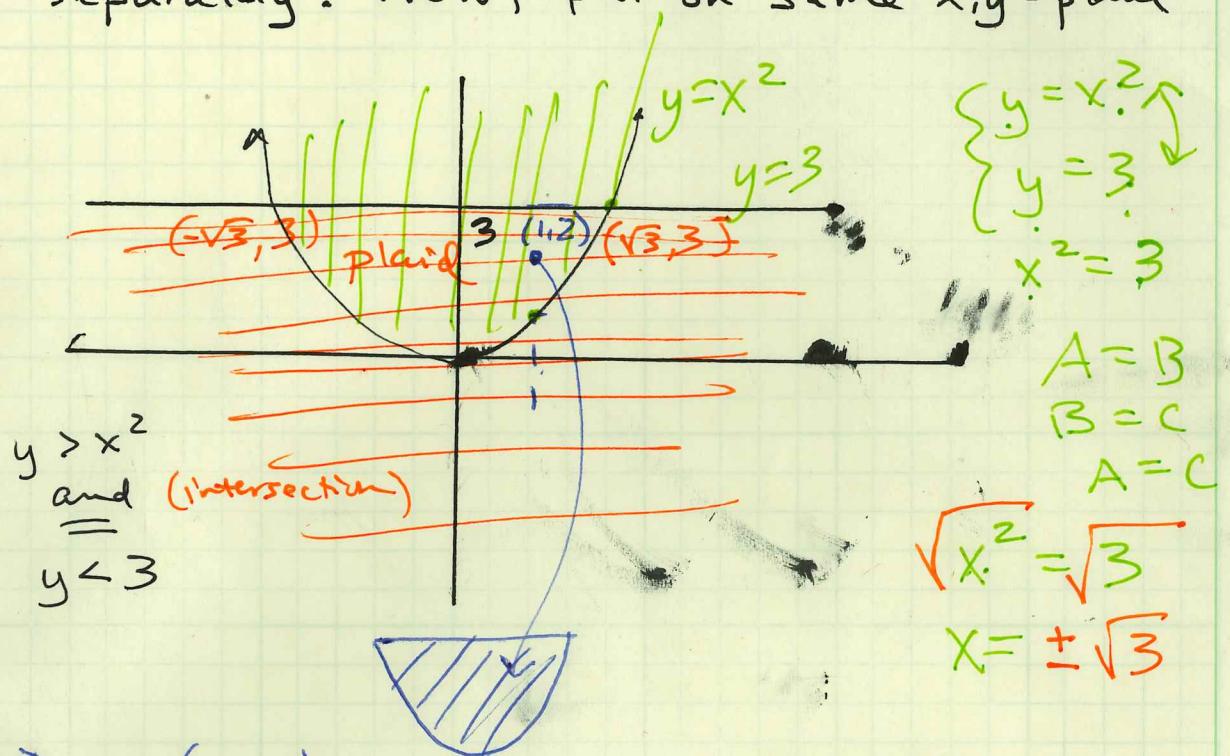
$$y = x + 0$$



x	$f(x) = \sqrt{x}$
0	$\sqrt{0} = 0$
1	$\sqrt{1} = 1$
2	$\sqrt{2}$
3	$\sqrt{3}$
4	$\sqrt{4} = 2$
9	$\sqrt{9} = 3$

When we graph fns, any vertical line drawn through the curve will intersect it only once, due to the uniqueness of y.

So far we've graphed lines & parabolas separately. Now, put on same x,y -plane



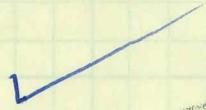
Does $(1, 2)$ satisfy both inequalities?

$$y \geq x^2$$

$$\checkmark \quad 2 \geq 1^2 = 1$$

$$y \leq 3$$

$$2 \leq 3 ?$$



not happen if on \leftarrow

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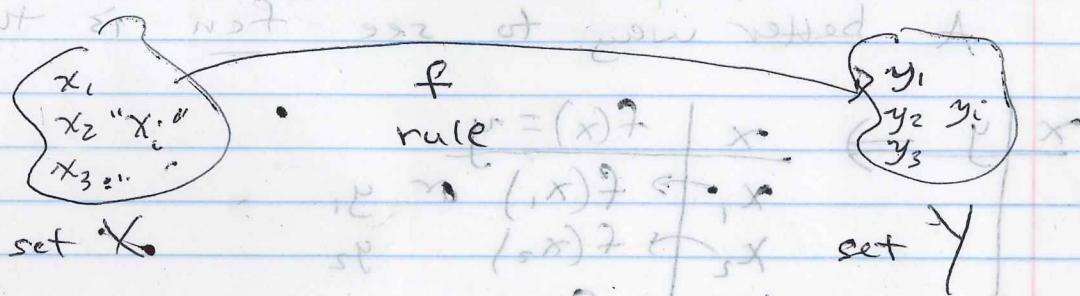
"Set" means a collection
of objects "elements"

* Definition of "function":

A function consists of 2 sets, X, Y

and a rule that assigns each element,
~~not to many~~, in set X to a unique element, y , in Y .

A good way to remember the def. of fcn
is this cartoon.



" y is a function of x "

is given by $y = f(x)$

" $f(x)$ " = " f of x "

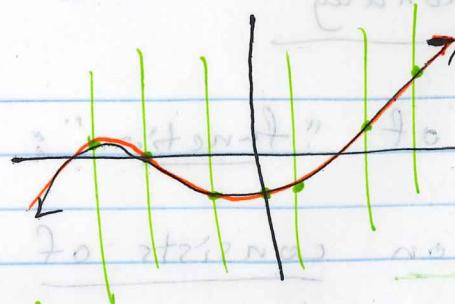
Def. A function ~~is~~ consists of 2 sets,

X and Y , and a rule $f(x)$ that assigns
(sends) each $x \in X$ to a unique $y \in Y$.

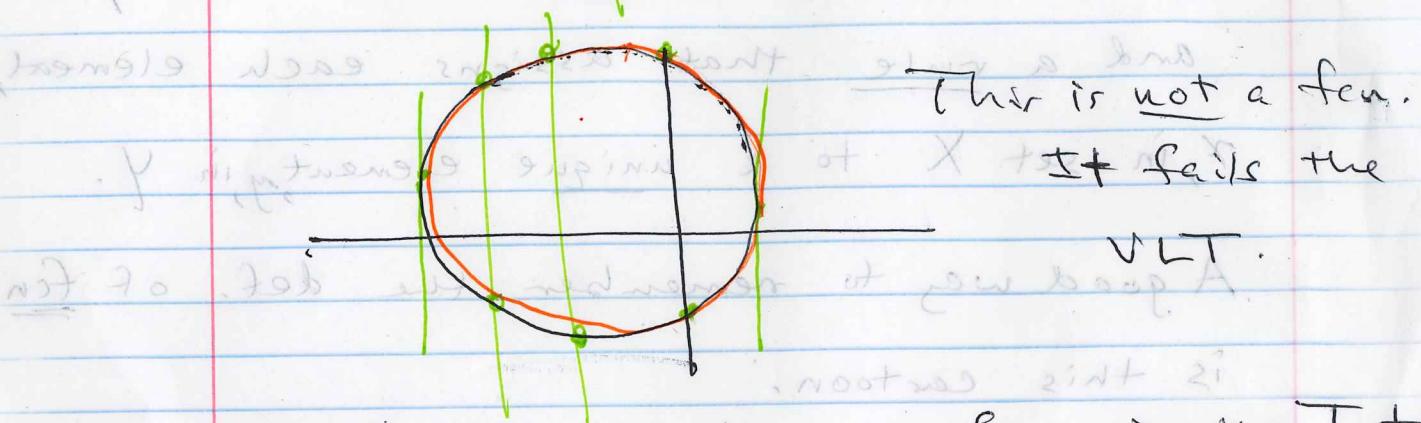
Functions, when graphed, pass the
vertical line test (VLT). That is,
because of the uniqueness of y 's,
any vertical line through graph intersects
the curve only once.

functions is "unique" $y \Rightarrow$ no y is repeated for
any x .

This is a y, x function!



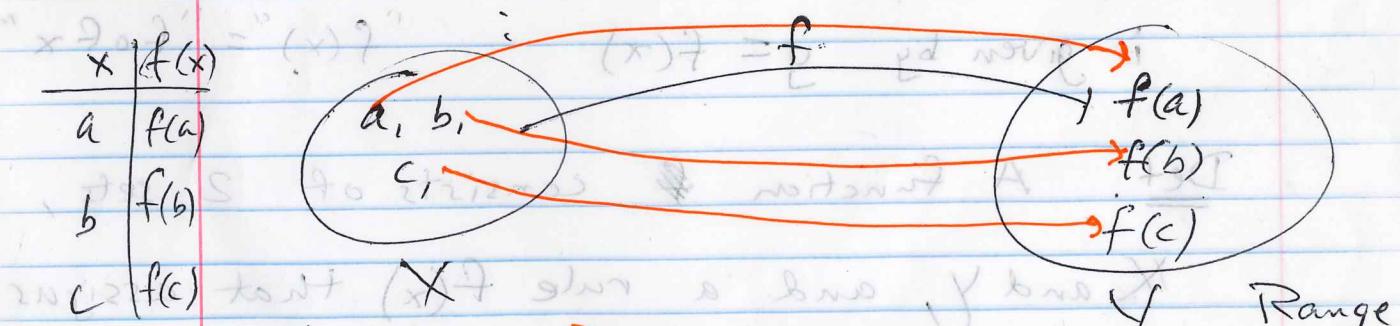
Passes the VLT.



This is not a function.

A better way to see if a function is the T-table.

x	$y \Rightarrow$	x	$f(x) = y$
		x_1	$f(x_1)$ or y_1
		x_2	$f(x_2)$ y_2
		x_3	$f(x_3)$ y_3



Domain - Determining this is a

big deal

Ex Consider lines $y = mx + b$, $f(x) = mx + b$

before

now

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(x) \rightarrow y where $x \in X$ to $y \in Y$

(x) \rightarrow y is a "function" of x

Function - consists of 2 sets, X, Y , and a rule that assigns each element $x \in X$ to a unique element $y \in Y$. (No y is repeated) set Y .

"Set" is a collection of objects.

!! two # "elements"

\in "element of", $x \in X$.

$\in + xS = (x)$ x is an element of set X

$f = \in + (z)S = (z)$

"y is a function of x" = (o)

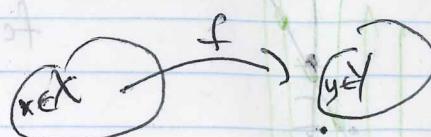
"y is a function of x" "f of x"

$y = f(x)$

* $f(x)$ is just another notation for y , but it describes the thing better

Before $y = mx + b$,

Now $f(x) = mx + b$



x	$f(x) = y$
-----	------------

Given the rule f ,

say $f(x) = 2x + 3$

we can fill in some ordered pairs (x, y) to get

$(1, 5), (2, 7), (3, 9), \dots$

Def

of a
Function

Domain - set of x for which $f(x)$ is "defined", meaning $f(x)$ will be a real number.

" D_f "

Ex

$$f(x) = 2x + 3$$

$D_f: x \in \mathbb{R}$

$(-\infty, \infty)$

For lines, parabolas, and any polynomial, the domain D_f is all $x \in \mathbb{R}$ reals.

Any real $\overset{x}{\text{#}}$ into $f(x)$ gives a real $\overset{f(x)}{\text{#}}$ out !!

y

"To trans"

trans is not x

" x to $f(x)$ "

trans is not x

D_f
of

\mathbb{R}

$$x | f(x) = 2x + 3$$

$$-5 | f(-5) = 2(-5) + 3 = -7$$

$$0 | f(0) = 2(0) + 3 = 3$$

$$6 | f(6) = 2(6) + 3 = 15$$

Graph

$(-5, -7)$

$(0, 3)$

$(6, 15)$

$$d + xw = (x)f \quad \text{with}$$

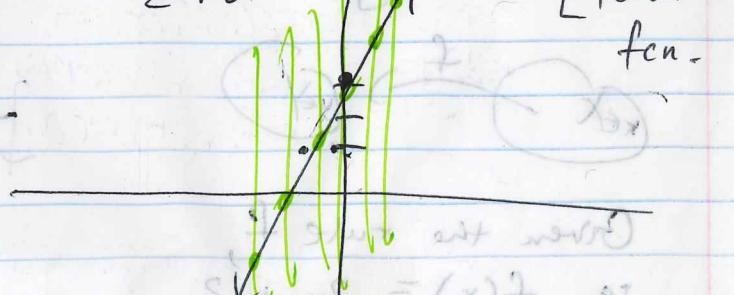
R positive
negative
zero

positive

linear
fcn.

$$y = (x)f$$

zero



$\mathbb{R} = \mathbb{Q} \cup \mathbb{R}'$ where \mathbb{R}'
 $\mathbb{R}' = \text{new business set}$

Video 7) Domain & range

$$f(x) = x + 1 \quad f(x) = \sqrt{x} \quad f(x) = \frac{1}{x}$$

$$D_f: (-\infty, \infty)$$

Because it's a polynomial, so
any real no. in
gives a real no.

$$D_f: [0, \infty)$$

Because negative
radicands do not
give real no.

$$f(-1) = \sqrt{-1} = i \notin \mathbb{R}$$

$x > 0$

$$D_f: x \neq 0$$

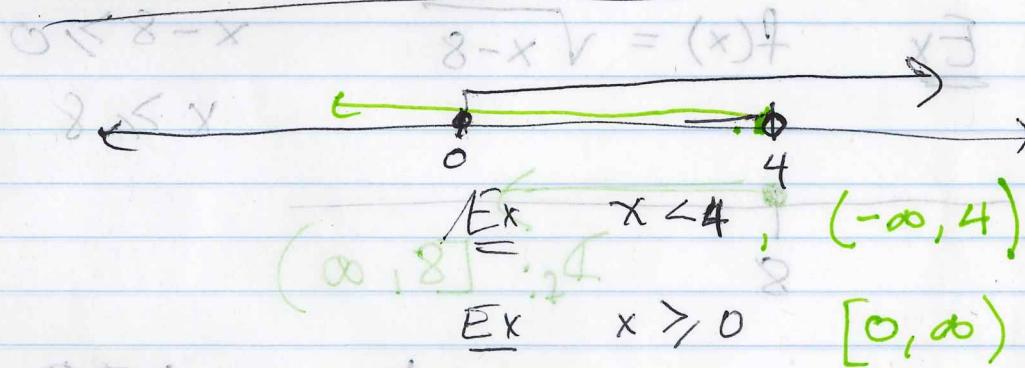
$$(-\infty, 0) \cup (0, \infty)$$

Because

$\frac{1}{0}$ is
not
defined.

$$(\infty, 1) \cup (1, -1) \cup (-1, \infty)$$

Review interval notation



Domain - How do we determine the set X

that comprises D_f ? (Watch domain

video slowly + carefully.) Ask yourself:

"What would make the denominator = 0?"

"What would ~~make~~ make the radicand < 0 ?"

The answers →

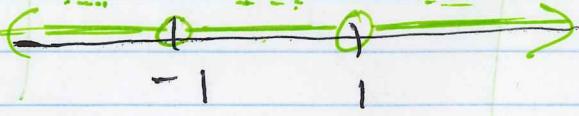
If there's no x in denominator or in the radicand then $D_f = \mathbb{R}$.

The answer to these tell us what

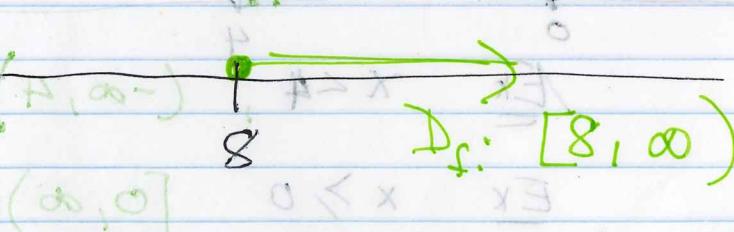
$$\frac{1}{x} = (x)^{-1} \text{ to leave out } 0 \text{ of } D_f, x = (x) \neq 0$$

Ex $f(x) = \frac{2}{x-7}$ we need $x-7 \neq 0$
 $x \neq 7$
 $(-\infty, 0) \cup (0, \infty)$
 $D_f: (-\infty, 7) \cup (7, \infty)$

Ex $f(x) = \frac{5}{x^2 - 1}$ we need $x^2 - 1 \neq 0$
 $x^2 \neq 1$
 $\sqrt{x^2} \neq \sqrt{1}$
 $x \neq \pm 1$


$$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

Ex $f(x) = \sqrt{x-8}$ we need $x-8 \geq 0$
 $x \geq 8$


$$D_f: [8, \infty)$$

Ex $f(x) = \frac{1}{\sqrt{x}}$ we need $x \neq 0, x > 0$
 $x > 0$
 $D_f: (0, \infty)$

Answers for (plusses + pluses = 96%)

" $x=0$ = rational numbers set below blow tank"

" $x > 0$ = non-zero set below blow tank"

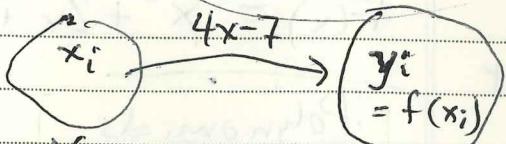


→ answer set

Ex $f(x) = 4x - 7$ from $y = 4x - 7$

x	$f(x) = 4x - 7$
-5	$-27 = f(-5)$
-1	$-11 = f(-1)$
0	$-7 = f(0)$
1	-3
2	1

Domain



X

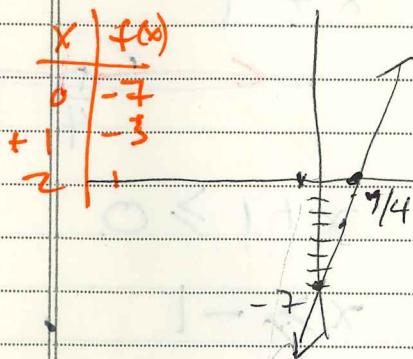
$$4x-7$$

X

$$4x-7$$

"x is an element of X"

x	$f(x)$
0	-7
1	-3
2	1



negative 0 positive

(x, y)

ordered pairs from table

(-5, -27), (-1, -11),

$$4x-7=0 \quad (0, -7) \quad (1, -3)$$

$$x = 7/4 \quad (2, 1)$$

To graph a fcn, we need to know its domain.

Def The domain of a fcn f is the set of real numbers (x) such that $f(x)$ is a real number. "If I give you an x -value, will you get a y -value that is a real #?"

How do we approach the way to determine domain?

Need to avoid $\frac{\#}{0}$, $\sqrt{-\text{negative}}$

"What cannot happen?"

Ex

Determine the domain of these funs.

What
x
values
will
give
me
zero
in a
denom
or
negative
under
radical
?

$$f(x) = x^2 + 2x + 1 \quad \text{All } x \in \mathbb{R} \text{ will}$$

give $f(x) \in \mathbb{R}$

Polynomials
have $D_f = \mathbb{R}$
 $(-\infty, \infty)$

$$f(x) = \frac{3}{x-4}$$

Here $x-4 \neq 0$

: So $x \neq 4$

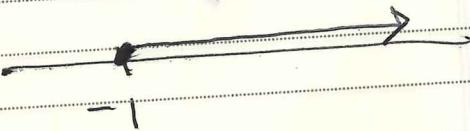
$$D_f: (-\infty, 4) \cup (4, \infty)$$



$$f(x) = \sqrt{x+1} \quad \text{Here, } x+1 \geq 0.$$

$$D_f: [-1, \infty)$$

$x \geq -1$

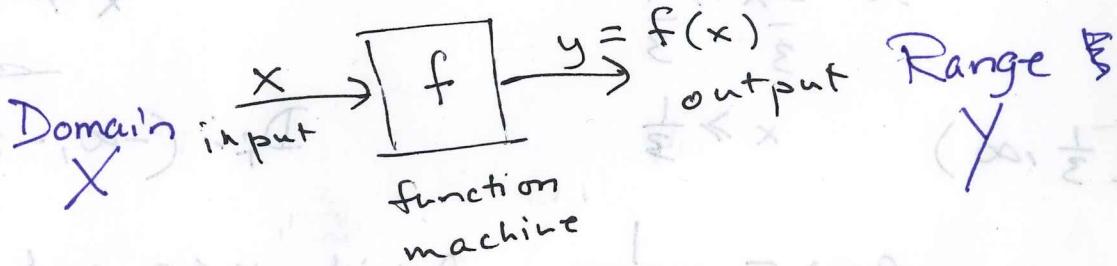


(1)

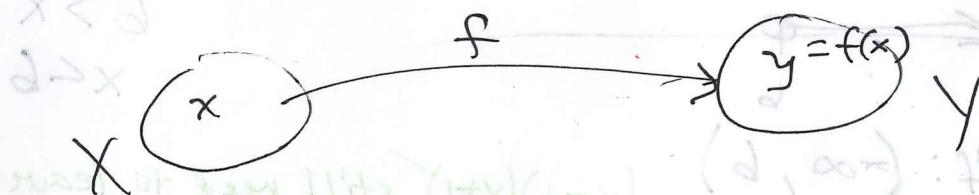
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Domain of function $f(x)$ -

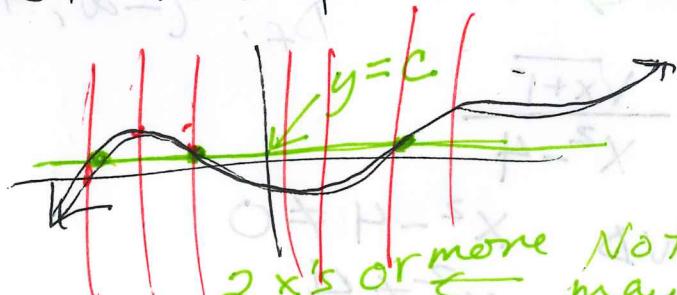
Def The domain of a fcn $f(x)$ is the set of x values for which the fcn is "defined". By "defined" we mean the input x value results in a real output y -value.



Def A function $f(x)$ consists of two sets, (X, Y) , (X being the domain, Y being the range,) and a rule that sends each $x \in X$ to a unique element $y \in Y$.



The graphical test that we have a fcn: if the graph of $f(x)$ passes the vertical line test, VLT



No 2 y -values for any given x .

2 x 's or more Note, horizontal line may intersect more than once!

may go to same y .

(2)

To determine the domain of a given fcn,
 avoid negatives ~~like~~ $\sqrt{}$, $\sqrt[3]{}$, $\sqrt[4]{}$.

Also, we avoid zero in denominator.

$$\underline{\text{Ex}} \quad f(x) = \sqrt{3x-1} \quad \begin{array}{l} \text{non-negative} \\ \text{radicand} \end{array}$$

$$\underline{\text{Ex}} \quad f(x) = \frac{5x}{x+2}$$

We need $3x-1 \geq 0$

so $3x \geq 1$

$\frac{1}{3} \cancel{(x)} \cancel{3} =$

$x \geq \frac{1}{3}$

$D_f: [\frac{1}{3}, \infty)$

$x+2 \neq 0$

$x \neq -2$

$D_f: (-\infty, -2) \cup (-2, \infty)$

$$\underline{\text{Ex}} \quad f(x) = \frac{1}{\sqrt{x}} \quad \begin{array}{l} \text{Avoid } x < 0 \text{ and } x = 0; x > 0 \\ D_f \end{array}$$

$$\underline{\text{Ex}} \quad f(x) = \frac{1}{\sqrt{6-x}} \quad \begin{array}{l} \text{Avoid } 6-x=0, \text{ and } 6-x > 0 \\ 6-x > 0 \end{array}$$

$-\infty \leftarrow \cancel{\Phi}$

b

$b < x$

$D_f: (-\infty, b)$ $(x-1)(x+1)$ still need to leave -1 out!

$$\underline{\text{Ex}} \quad f(x) = \frac{x^2-1}{x+1} \quad \begin{array}{l} \text{Need } x+1 \neq 0 \\ x \neq -1 \end{array}$$

$D_f: (-\infty, -1) \cup (-1, \infty)$

$$\underline{\text{Ex}} \quad f(x) = \frac{\sqrt{x+1}}{x^2-4}$$

$x+1 \geq 0$ AND $x^2-4 \neq 0$

$x \geq -1$

$x^2 \neq 4$



Compare D_f for $f(x) = \sqrt[3]{x+1}$ (2a)

to D_g for $g(x) = \sqrt[3]{x+1}$

$f(x)$ is an even root fn, so $x+1 \geq 0$

$g(x)$ is an odd root fn, so $x+1$ can be any IR

$$D_f: x+1 \geq 0 \rightarrow x \geq -1 \quad [-1, \infty)$$

$$D_g: x+1 \text{ can be any } \# \text{ in IR, then } D_g: (-\infty, \infty)$$

Ex Find D_f ; avoid negative radicand and zero in a denominator.

$$f(x) = \sqrt{\frac{x}{4} + 1}$$

Require $\frac{x}{4} + 1 \geq 0$

$$\frac{x}{4} \geq -1 \cdot 4$$
$$x \geq -4$$

$\xrightarrow{-4} [-4, \infty)$

By the way,
 $\sqrt{0} = 0 \in \text{IR}$

$$f(x) = \frac{x+3}{\sqrt{x}}$$

$\xrightarrow{0} D_f: (0, \infty)$

$\sqrt{x} \neq 0$ b/c it's in denom
 $x \geq 0$ b/c it's a radicand
 $x+3$ is good for all IR

$$g(x) = \frac{\sqrt{x-7}}{x^2 - 9}$$

Need both of the following to be true:
 $x-7 \geq 0$ and $x^2 - 9 \neq 0$

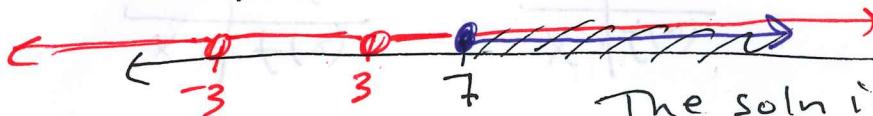
$$x-7 \geq 0$$

$$x \geq 7$$

$$x^2 - 9 \neq 0$$

$$x^2 \neq 9$$

$$x \neq 3, -3$$



The soln is the overlap $[7, \infty)$

25)

$$\text{Ex } f(x) = \frac{\sqrt{x+1}}{x^2 - 4}$$

$$\begin{aligned} x+1 &\geq 0 \\ -1 & \leq -1 \\ x &> -1 \end{aligned}$$

$(-\infty, -1]$

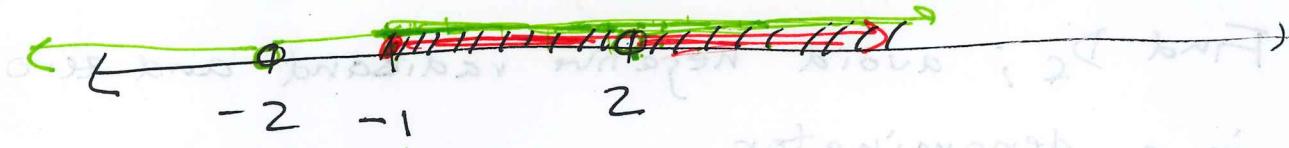
$$x^2 - 4 \neq 0$$

$$\checkmark x^2 \neq 4$$

$$x \neq 2, -2$$

$$x < 1 + x : \cancel{x} \quad -2 \quad 2$$

$(-\infty, -1] \cup (-1, 2) \cup (2, \infty)$



$$D_f: [-1, 2) \cup (2, \infty)$$

$$\sqrt{1+x} = (x)^{\frac{1}{2}}$$

$$P: 1 - \sqrt{\frac{x}{x}} = 0$$

$$H: \sqrt{x}$$

$$(\infty, \infty)$$

Δ

want $1+x \geq 0 \neq x^2$

$$\frac{1+x}{x^2} = (x)^{\frac{1}{2}}$$

possible with $x \geq 0$

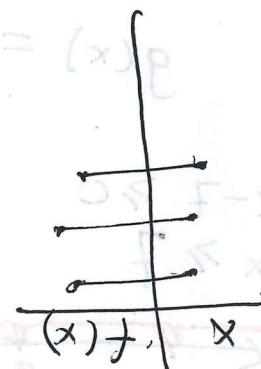
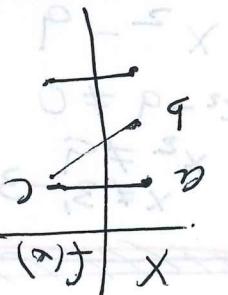
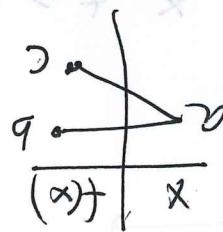
No root for $x < 0$

$$(\infty, 0) : \Delta$$

allow of $x \geq 0$ to add less

$0 \neq P-x$ lens $0 \leq P-x$

\bar{C}



(3)

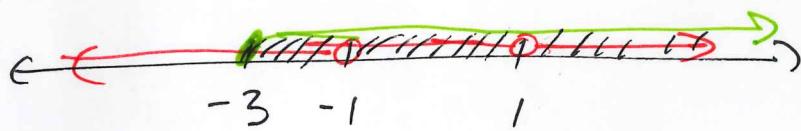
Include any x
 \rightarrow where graph
has 2 solid
lines.



$$(-1, 2] \cup (2, \infty)$$

EY $f(x) = \frac{\sqrt{x+3}}{x^2-1}$

$$\begin{aligned} x+3 \geq 0 &\rightarrow x \geq -3 \\ x^2-1 \neq 0 &\rightarrow x^2 \neq 1 \\ &\rightarrow x \neq \pm 1 \end{aligned}$$



$$D_f: [-3, -1) \cup (-1, 1) \cup (1, \infty)$$

Thurs

$$f(x) = \sqrt{x^2-1} \quad \underbrace{x^2-1 \geq 0}$$

Video

9/13 /
10:07

Review graph inequalities

(1) (8)

$$\begin{cases} y > -6 \\ y \leq \frac{1}{2}x - 5 \end{cases}$$

1. Treat them like equations

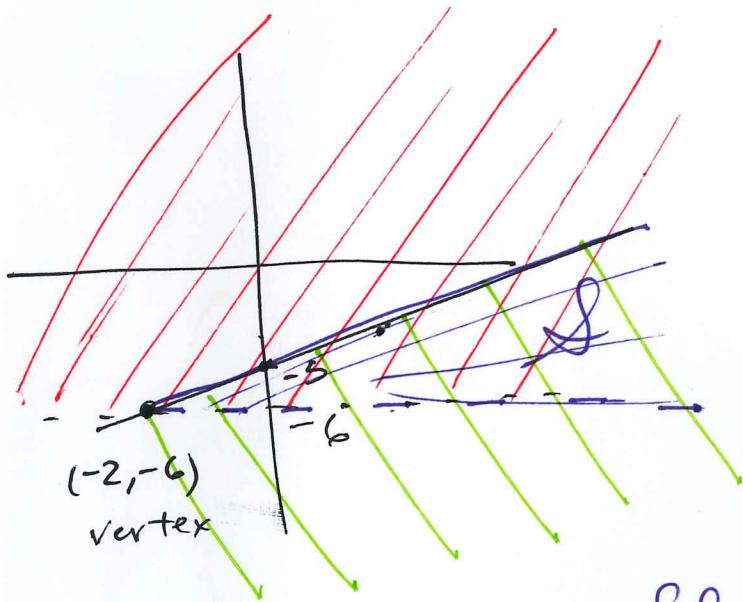
2. Use dashed line for strict inequality $<$, $>$

3. Shade the appropriate regions (check a pt, esp $(0, 0)$)

4. Find vertex (intersection) as usual for system.

5. ~~Show~~ Label the soln set

SS - All (x, y) in this region satisfy both inequalities.



Fact Polynomials have no restrictions on x values.
Therefore, $D_{\text{poly}}: (-\infty, \infty)$

Fact $\frac{1}{f(x)}$ has restriction $f(x) \neq 0$

Fact $\sqrt{f(x)}$ has restriction $f(x) \geq 0$

Let's read domains off given graph!

$$f(x) = |x|$$

Graph shows points $(-1, 1)$, $(0, 0)$, $(1, 1)$, and $(4, 4)$.
 $D_f: (-\infty, \infty)$

$$f(x) = ax^n + an-1 \cdot x^{n-1} + \dots + a_0$$

Graph shows a curve passing through $(-1, 1)$ and $(1, 1)$.
 $D_f: (-\infty, \infty)$

$$f(x) = \sqrt{x}$$
$$f(x) = \frac{1}{x}$$

Graph shows $f(x) = \sqrt{x}$ for $x \geq 0$ and $f(x) = \frac{1}{x}$ for $x < 0$.
 $D_f: [0, \infty)$
 $(-\infty, 0) \cup (0, \infty)$