Week 3 — Sept 6 — Math 106

More absolute value, exponents, radicals

First, reduce fractions before multiply!

Quiz

$$\frac{\cancel{9}^{3}}{\cancel{10}_{1}}\cdot\frac{\cancel{17}^{1}}{\cancel{12}_{1\,\cancel{4}}}\cdot\frac{\cancel{80}^{\cancel{8}\,8}}{\cancel{34}_{\cancel{2}\,1}} = \frac{3\cdot1\cdot1}{1\cdot1\cdot1} = 3$$

other skill from quiz: mixed # ⊕, ⊖

ex $8\frac{1}{3}\cdot\frac{3}{3} + 5\frac{2}{9} = 13\frac{5}{9}$; ex $8\frac{1}{3}\frac{3}{3} - 5\frac{2}{9} = 8\frac{3}{9} - 5\frac{2}{9}$

$= 3\frac{1}{9}$

ex $8\frac{2}{9} - 5\frac{1}{3}$

$= 8\frac{2}{9} - 5\frac{3}{9}$ Don't make improper fractions

Or this also, $\frac{2-3}{9} = \frac{-1}{9}$

Don't do this! $3\frac{-1}{9}$ means

$3 - \frac{1}{9}$

$= 2\frac{8}{9}$

Instead $\overset{7+1}{8}\frac{2}{9}$

$$\begin{array}{r} 8\frac{2}{9} \\ -5\frac{3}{9} \\ \hline \end{array}$$

$$= \begin{array}{r} 7+1\frac{2}{9} \\ -5\frac{3}{9} \\ \hline \end{array} = \begin{array}{r} 7\frac{11}{9} \\ -5\frac{3}{9} \\ \hline 2\frac{8}{9} \end{array}$$

Mental math, not calculators, to perceive magnitude automatically.

Ex $0.001 = \frac{1}{1000} = .1\%$

Ex $\frac{18}{1/3} = 54$ # of $\frac{1}{3}$'s in 18

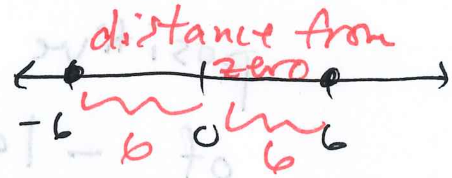
$18 \div \frac{1}{3} = 18 \cdot \frac{3}{1}$

Ex ( )( )

Absolute value – by definition, graphically*, algebraically**, interval $\lesssim$

(1) **Ex** $|a| = 6$ means the a values 6 units from either side of 0.

distance from zero



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Algebraically, $a = -6$ or 6 **

Def $$|a| = \begin{cases} a \text{ itself}, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Ex $$|-14| = \begin{cases} -(-14) = 14 \\ \end{cases}$$

Later, with algebraic notation:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

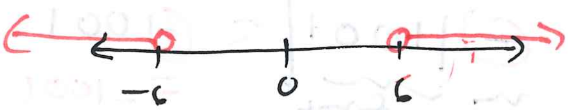
(2) **Ex** $|a| \leq 6$

graphic

means all values within 6 units, and at 6 units from zero.

of,

(3) **Ex** $|a| > 6$, $a > 6$ or $a < -6$,



$-6 \leq a \leq 6$ algebraic

$[-6, 6]$ interval

(later, Thurs)

What if $-|a|$?

This is $-a$, since $|a|$ is always positive, but $(-)$ is later. Think of $-|a|$ as $(-1)|a|$. Think "$|\ |$" as parentheses (do first)

<u>Ex</u> $-|7| = -7$

<u>Ex</u> $-|-.3| = (-)(.3) = -.3$

<u>Recall</u> $|a||b| = |a \cdot b|$ (equivalent), <u>ex</u> $|-4||2|$ $= |-4 \cdot 2|$ $= |-8| = 8$ $\quad 4 \cdot 2 = 8$

But $|a| + |b| \geq |a+b|$

Why? We need only one example to show $|a| + |b| \neq |a+b|$

Counter-example:
Let $a = -1$, $b = 5$.
Then $|-1| + |5| = 1 + 5 = 6$ $\quad 6 \neq 4$
and $|-1+5| = |4| = 4$

<u>Ex</u> $|-8| = 8$, $|-7264| = 7264$

$|1001| = 1001$ but $-|1001| = -1001$ first $= -1001$

Exponents, Radicals, Properties/Rules/Facts

base; exponent, $n \in \mathbb{R}$

Def $a^n =: a \cdot a \cdot \cdots a$ n factors of a , a^n "nth power of a"

ex $2^4 = 2\cdot2\cdot2\cdot2 = 16$

1. $a^n + a^m$ is in simplest, final form

ex $4^2 + 4^3$ done; $x^2 + x^3$ done, cannot combine

2. $a^n a^m = a^{n+m}$

$\underbrace{a \cdot a \cdots a}_{n \text{ factors}} \; \underbrace{a \cdot a \cdots a}_{m \text{ factors}} \rightarrow n+m$ factors

3. $\frac{a^n}{a^m} = a^{n-m}$, $\frac{\overbrace{a \cdot \cancel{a} \cdot \cancel{a} \cdots a}^{n}}{\underbrace{\cancel{a} \cdot a \cdots \cancel{a}}_{m}} = a^{n-m}$ remain

ex $\frac{6^4}{6^5} = 6^{4-5} = 6^{-1} = ?$

4. $a^{-n} = \frac{1}{a^n}$, $6^{-1} = \frac{1}{6}$

5. $(a^n)^m = a^{nm}$, ex $(5^2)^3 = 5^2 \cdot 5^2 \cdot 5^2$

$\downarrow$

$5^{2\cdot3} = 5^6$

~~$(a \cdot a\, a \cdots a)\, a\, a\, a \cdots a$~~ n m

6. $a^{\frac{1}{n}}$ means $\sqrt[n]{a}$; $a^{1/2}$ means $\sqrt{a}$

$a^{1/3}$ means $\sqrt[3]{a}$, for $n > 2$, we need to write n in the "index" position of radical

* Facts $a^1 = a$, $a^0 = 1$, except 0^0 which is not defined

ex $17^1 = 17$

$17^0 = 1$

0^0 undefined

Text Cumulative Test A first half

#1 -2π only irrat'l

#2

method 1

$$\frac{\frac{5\cdot 2}{5\cdot 3} - \frac{3\cdot 1}{3\cdot 5}}{\frac{2}{2}\cdot\frac{1}{2} + \frac{1}{4}} = \frac{\frac{7}{15}}{\frac{3}{4}} = \frac{7}{15}\cdot\frac{4}{3} = \frac{28}{45} \quad //$$

method 2

LCD of $3, 5, 2, 4 = 3\cdot 5\cdot 2^2 = 60$

$(2\cdot 2) = 2^2$

$$\left(\frac{\frac{2}{3} - \frac{1}{5}}{\left(\frac{1}{2} + \frac{1}{4}\right)}\right)\cdot\left(\frac{60}{60}\right) = \frac{40 - 12}{30 + 15} = \frac{28}{45} \quad //$$

#3 $\dfrac{2(x-y)^{-1}}{-1(y-x)}$ recall $\dfrac{a-b}{\underbrace{b-a}_{-1(a-b)}} = -1$

$= +2$

$$-(\bigcirc - \heartsuit) = \frac{\heartsuit - \bigcirc}{\cancel{\bigcirc - \heartsuit}} = -1$$

$\dfrac{2(x-y)}{-(y-x)}$ $x=-2$ $y=-1$

$$\frac{2(-2 - -1)}{-(-1 - -2)}$$

$$= \frac{2(+1)}{-1(1)} = 2$$

#3a $\dfrac{-3}{4}\cdot\dfrac{b-c}{\cancel{c-b}}^{-1} = \dfrac{3}{4}$

#4 $\quad \frac{\cancel{2}^{1}}{1}\left(\frac{x-y}{\cancel{4}\,2}\right) + \frac{y-x}{2} = \frac{x-y}{2} + \frac{y-x}{2} = \frac{\cancel{x}-\cancel{y}+\cancel{y}-\cancel{x}}{2}$

$= 0$

Consider $\quad \frac{x-y}{2} + \frac{y-x}{2}$

reverse

$$= \frac{x-y}{2} - \frac{(x-y)}{2} = 0$$

Aside $\quad a - a = 0 \; ; \quad a + (-a) = 0 \leftarrow$

$\frac{a}{a} = 1 = a \cdot \frac{1}{a} = 1 \leftarrow$ multiplicative identity $a \cdot 1 = a$

number $\quad a$

$\uparrow$ reciprocal of number

5. $$\frac{-|2-5|}{|-5|-|-2|} = \frac{-|-3|}{5-2} = \frac{-3}{3} = -1$$

True or false $\quad |a| + |b| = |a+b|$ for all a, b

False ; counterexample, let $a = -4$, $b = 7$

$$|-4| + |7| \stackrel{?}{=} |-4+7|$$

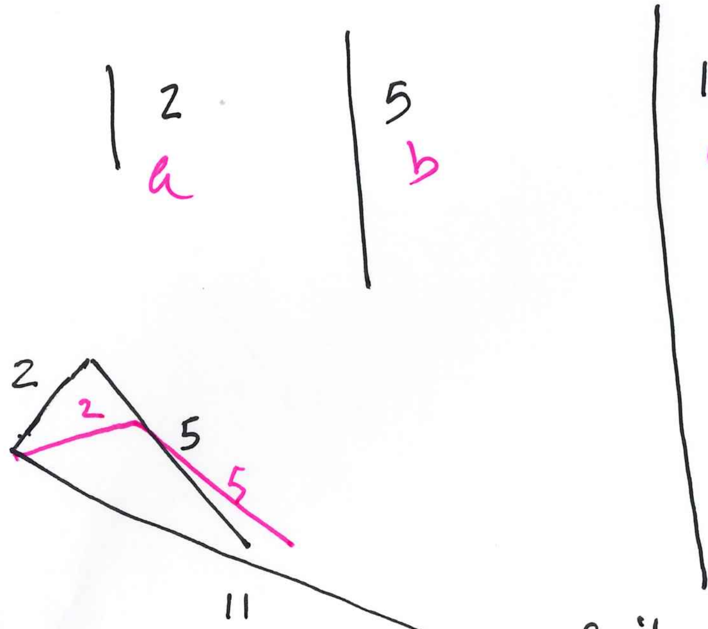
$$4 + 7 \qquad |3|$$

$$11 \neq 3$$

Triangle Inequality $\quad |a| + |b| \geq |a+b|$

It's = if $a, b > 0$ or $a, b < 0$

Triangle inequality via (by way of) geometry.



Join these so we get closed triangle

Can't do it!

Because the sum of any lengths that we wish to make a triangle from have to, greater than the third

fail

$a+b>c$, ~~a+b~~ $\boxed{2+5=7<11}$

$a+c>b$ $\quad 2+11=13>5$

$b+c>a$ $\quad 5+11=16>2$

Levi 5, 2, 4

Amanda 5, 5, 9

Rane 3, 4, 5

~~Emily~~ 2, 3, 5
Emely

$$|a|+|b| \geq |a+b|$$

$$2+3=5$$

1

#2

(1) $\dfrac{\frac{2}{3}-\frac{1}{5}}{\frac{1}{2}+\frac{1}{4}}$ $\dfrac{\frac{10-3}{15}}{\frac{2+1}{4}} = \dfrac{\frac{7}{15}}{\frac{3}{4}} = \frac{7}{15}\cdot\frac{4}{3} = \frac{28}{45}$

(2) LCD of 3, 5, 2, 4 is 60

3 1 5 1 2 1 2 2

2^2

$$\frac{60\left(\frac{2}{3}-\frac{1}{5}\right)}{60\left(\frac{1}{2}+\frac{1}{4}\right)}\cdot\frac{60}{60} = \frac{40-12}{30+15} = \frac{28}{45}$$

$$\frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{10}-\frac{1}{9}}\quad \frac{180}{180} = \frac{60-45}{18-20} = \frac{15}{-2} = -\frac{15}{2}$$

10 9 3 22

25 33 4

$5^1\cdot3^2\cdot2^2$ $\frac{45}{4}$

#3 $\cancel{2\left(\frac{x-y}{4}\right)+\left(\frac{y-x}{21}\right)}$

#3 $\dfrac{2(x-y)^{-1}}{-1(y-x)}$ when $x=-2$ $y=-1$ $\dfrac{-2}{-1}=2$

$$\frac{2(-2--1)}{-1(-1--2)} = \frac{2(-1)}{-1} = 2$$

SCRAP PAPER

2

More $\frac{a-b}{b-a} = -1.$ $\qquad \frac{8\,\cancel{(2-m)}\,\cancel{(m+3)}^{\,1}}{2\,\cancel{(3+m)}\,\cancel{(m-2)}_{\,-1}} = -4$

Use of this: $f(x) = \frac{x^2-9}{x+3} = \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}}$

hole at $x=-3$

$g(x) = \frac{1-x^2}{\cancel{1+x}\ \ x-1} = \frac{\cancel{(1-x)}(1+x)}{\cancel{(x-1)}_{\,-1}} = -(1+x)$

#4 $\cancel{2}\left(\frac{x-y}{\cancel{4}_{\,2}}\right) + \frac{y-x}{2} = \frac{(x-y)}{2} + \frac{(y-x)}{2}$

$\frac{\cancel{x}-\cancel{y}+\cancel{y}-\cancel{x}}{2} = \frac{0}{2} = 0 = \frac{x-y}{2} + \frac{-(x-y)}{2} = 0$

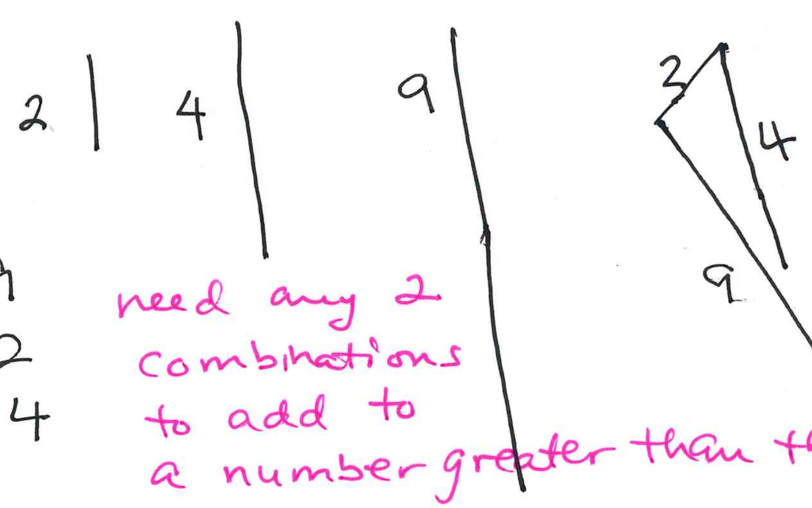
#5. $\frac{-|2-5|}{|-5|-|-2|} = \frac{-|-3|}{5-2} = \frac{-3}{3} = -1$

Aside
$|a+b| = |a|+|b|$
$|a+b| > |a|+|b|$
$*\ |a+b| \le |a|+|b|$ True for all a, b

Triangle inequality

Geometric construction

Aside



never close into triangle

$4+2 < 9$
$4+9 > 2$
$2+9 > 4$

need any 2 combinations to add to a number greater than the third

#6. $5|a||b| - |ab|$ $\quad a = -2 \quad b = -5$

$5|ab| - 1|ab| = 4|ab| = 4 \cdot 10 = 40$

* $\left(m^{-1} + n^{-1}\right)^2$ $\qquad (a+b)^n \neq a^n + b^n$

$= \left(\frac{n \cdot 1}{n \cdot m} + \frac{m \cdot 1}{mn}\right)^2 = \left(\frac{1}{m} + \frac{1}{n}\right)\left(\frac{1}{m} + \frac{1}{n}\right)$

$= \left(\frac{n+n}{mn}\right)^2$

Section 8.3 Roots + Radicals

Def $\boxed{a^{1/n} = \sqrt[n]{a}}$ "the nth root of a"

Exs $9^{\frac{1}{2}} = \sqrt{9} = 3$, $\left(\frac{1}{4}\right)^{1/2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

so $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

backwards $\sqrt{\frac{1}{4}} = \frac{1}{2}$

Exs $8^{1/3} = \sqrt[3]{8} = 2$ $\qquad 8^{2/3} = \left(8^{1/3}\right)^2$

$= 2^2 = 4$

Note: If $a^{\frac{1}{n}}$ has $n = 2$, the $\sqrt{a}$ has no "2" in the corner (index)

4

(a) $5^{\ } = 125$ (b) $(-3)^{\ } = (-3)^{\ } = \frac{1}{-3} = -\frac{1}{3}$

5^{7-4}

$\frac{9}{-27} = -\frac{1}{3}$

(d) $\frac{-(\heartsuit)^6}{(\heartsuit)^8} = -\heartsuit^{-2} = \frac{-1}{\heartsuit^2} = \frac{-1}{(2y-1)^2}$

(e) $\frac{x^{2n+1}}{x^n} = x^{2n+1-n}$

$= x^{n+1}$

(f) $\frac{(x^3)^4}{x^7 (x^2)^5} = \frac{x^{12}}{x^7 \cdot x^{10}} = \frac{x^{12}}{x^{17}} = x^{-5} = \frac{1}{x^5}$

$(-a)^n \neq -a^n$

$\|$ means

$-(a^n)$

Ex $32^{3/5} = (32^{1/5})^3 = \left(\sqrt[5]{32}\right)^3$

Powers of 2 – Know them!!!

1 2 4 8 16 32 64 128 256 512

2^5 2^4 2^9 2^7 2^6 2^3 2^0 2^{10} 2^1 2^2 2^8

$2\cdot 2^4$ $2\cdot 2^6$ 11

1024

Facts: Even radical roots are always positive.
Only positive numbers have even roots

Odd roots can be positive or negative
Negative numbers have no even roots

Ex $\sqrt[2]{1} = 1$, $\sqrt[2]{-1}$ DNE, $\sqrt[3]{-1}$ -1 ; $\sqrt{64}$ 8, $\sqrt{-64}$ DNE, $\sqrt[3]{-64}$ -4

$\cancel{\sqrt{16}\ \sqrt{-16}\ \sqrt[3]{1}}$

$-1 \cdot -1 \cdot -1 = -1$

$(-1)^3 = -1$ hence $\sqrt[3]{-1} = -1$

$(-4)(-4) = 16$, $16(-4) =$

Sec 8.3

Deal with neg exp first

6

1. $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$ /

2. $4^{5/2} = (4^{1/2})^5 = 2^5 = 32$ /

3. $(-125)^{-1/3} = \left(\frac{1}{-125}\right)^{1/3} = \frac{1^{1/3}}{(-125)^{1/3}} = \frac{1}{-5} = -\frac{1}{5}$ /

4. $(-64)^{-2/3} = \left(-\frac{1}{64}\right)^{2/3} = \left(\left(-\frac{1}{64}\right)^{1/3}\right)^2 = \left(-\frac{1}{4}\right)^2$

$= \frac{1}{16}$ /

5. $\left(\frac{-8}{125}\right)^{2/3} = \left[\left(\frac{-8}{125}\right)^{1/3}\right]^2$ or $\cancel{\left[\left(\frac{-8}{125}\right)^2\right]^{1/3}}$

too hard

Tip - take root before the power

$a^{m/n}$ ← power, ← root

$\left(\frac{-2}{5}\right)^2 = \frac{4}{25}$ /

25. $\left(\frac{x^{18}}{y^{-6}}\right)^{2/3} = \frac{x^{18 \cdot \frac{2}{3}}}{y^{-6 \cdot \frac{2}{3}}} = \frac{x^{12}}{y^{-4}}$

bring exp in

30. $\left(\frac{125x^{12}}{y^{-3/2}}\right)^{-2/3} = \left(\frac{125^{-2/3}\, x^{\frac{12}{1} \cdot \frac{-2}{3}}}{y^{-\frac{3}{2} \cdot -\frac{2}{3}}}\right)$

$= \frac{5^{-2}x^{-8}}{y} = \frac{1}{5^2x^8y} = \frac{1}{25x^8y}$

Warning $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{(-2)(-2)(-2)} = \frac{1}{-8} = -\frac{1}{8}$

Common error $(-2)^{-3} \neq \frac{1}{2^3}$ Deal w̄ exponent
Deal w̄ value

Warning -1^n is not $(-1)^n$

Rather, it's $-(1^n)$

$\cancel{-1^n = -(1^n)}$ $-1^n \neq (-1)^n$

$-1^n = -(1^n)$

Ex $-3^4 \overset{\text{means}}{=} -(\underbrace{3^4}_{3^2\cdot 3^2}) = -81$

$(-3)^4 = (-3)(-3)(-3)(-3) = 81$

Ex $-|a|$ ~~might~~ is always negative

$|a|$ always positive

Book $\frac{a^m}{a^n} = a^{m-n}$ ~~if $m > n$~~

$= \cancel{\frac{1}{a^{n-m}}}$ ~~if $m < n$~~

Only rule needed for $\frac{a^m}{a^n} = a^{m-n}$. Deal with exp negative after:

Ex $$\frac{x^{-3}}{x^2} = x^{-3-2} = x^{-5} = \frac{1}{x^5}$$

Famous exponent problem on placement test:

$$\Big(\underbrace{\frac{1}{m}\cdot\frac{n}{n} + \frac{1}{n}\cdot\frac{m}{m}}_{LCD = mn}\Big)^{-1}$$

"Simplify" – leave no ( ) nor neg. exps.

$$= \left(\frac{n+m}{mn}\right)^{-1} = \frac{(n+m)^{-1}}{(mn)^{-1}} = \left(\frac{mn}{n+m}\right)^{1} \;//$$

Helpful shortcut $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$

Only works with × and ÷

Rules of radicals

$\sqrt{a}\sqrt{b} = \sqrt{ab}$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\underbrace{\sqrt{a+b} \text{ done} \quad , \quad \sqrt{a-b} \text{ , done}}_{\text{Do not separate}}$

Ex Simplify $\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$ //

Ex Simplify $\sqrt{x^2 y} = \sqrt{x^2}\sqrt{y} = x\sqrt{y}$ //

Ex Simplify $\sqrt{9+27x} = \sqrt{9(1+3x)} = \sqrt{9}\sqrt{1+3x}$

factor out 9

$= 3\sqrt{1+3x}$ //

$9\left(\frac{9}{9} + \frac{27}{9}x\right)$

$(1 + 3x)$

Ex Simplify

$\sqrt{100 + 1000x^4} =$

$= \sqrt{100(1+10x^4)}$

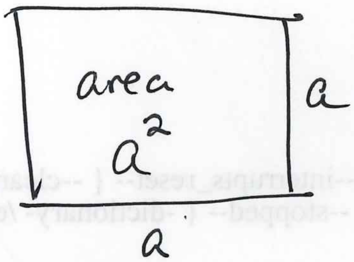
$= \sqrt{100}\sqrt{1+10x^4}$

$= 10\sqrt{1+10x^4}$ //

Sec 8.3 $\qquad a^{1/n}$ means $\sqrt[n]{a}$

That is, the number which raised to the nth power gives back a.

Book



$\sqrt{a^2} = a$

By tradition we don't write "2" in $\sqrt{a}$ for square roots.

Ex $\quad 4^{1/2} = \sqrt{4} = 2 \qquad$ "(square) root 4"

"radical 4"

"$\sqrt{}$" radical symbol

But $\quad 64^{1/3} = \sqrt[3]{64} = 4 \quad$ b/c $\quad 4 \cdot 4 \cdot 4 = 64$

Memorize $\quad 2^n, \quad 3^n, \quad 4^n, \quad 5^n \quad$ upto $n = \ldots$

$n=10 \quad n=5 \quad n=4 \quad n=4$

$2^7 = 2^4 \cdot 2^3$
$= 16 \cdot 8$
$= 128$

$5^3 = 5^2 \cdot 5 = 25 \cdot 5 = 125$

$$a^{m/n} = \underbrace{(a^{1/n})^m}_{\text{prefer to do like this}} \quad \text{or} \quad (\underbrace{a^m}_{})^{1/n}$$

$$\left(\text{giant } \#\right)^{1/n}$$

Ex $32^{4/5} = \cancel{32}\ (32^4)^{1/5}$

$= (32^{1/5})^4 = 2^4 = 16$

Def $a^{m/n}$ means $\sqrt[n]{a^m}$ or $\underbrace{(\sqrt[n]{a})^m}_{\text{better}}$

Friday 9/9/22 Trouble spots in exponent/radical simplification

Why exponents? Sciences - half-life in chemistry, in physics, Bacterial growth, in bio, ennviron, rxns in chem, Monetary growth and loss in econ.

Exponents $\longleftrightarrow$ Radicals

$a^{1/n} \longleftrightarrow \sqrt[n]{a}$

$a^{m/n} = (a^m)^{1/n}$ or $\underbrace{(a^{1/n})^m}_{\text{better}} \longleftrightarrow \sqrt[n]{a^m}$ or $\underbrace{(\sqrt[n]{a})^m}_{\text{better often}}$

We Know $a^m + b^m \neq (a+b)^m$

$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$

that is, $a^{1/2} + b^{1/2} \neq \sqrt{a+b} \neq (a+b)^{1/2}$

Freshman error

Consider $(-1)^{4/5} = \Big(\underbrace{(-1)^{1/5}}_{\text{fifth root}}\Big)^{4 \text{ th power}}$ $\quad (-1)\ \underbrace{(-1)(-1)}_{1}\ \underbrace{(-1)(-1)}_{1}$ $\quad -1 \cdot 1 \cdot 1$

$= (-1)^4 = 1$

$((-1)^4)^{1/5} = 1^{1/5} = 1$

Is $-1^{4/5} = (-1)^{4/5}$? no

$(-1^4)^{1/5}$ means $(-(1)^4)^{1/5} = (-1)^{1/5} = -1$

augers well
good sign

Sec 8.3

position in fraction, no effect on sign of base

#3 $(-125)^{-1/3} =$

Know $b^{-n} = \frac{1}{b^n}$; $(-125)^{-1/3} = \left(\frac{1}{-125}\right)^{1/3}$

$$= \frac{(1)^{1/3}}{(-125)^{1/3}} = \cancel{+}\frac{1}{-5} = \boxed{-\frac{1}{5}}$$

#4 $(-64)^{-2/3}$ first, deal with neg exp

$$\to \left(\frac{1}{-64}\right)^{2/3} = \frac{1^{2/3}}{(-64)^{2/3}} = \frac{1}{[(-64)^{1/3}]^2}$$

$\sqrt[3]{(-64)(-64)}$

$(\sqrt[3]{-64})^2$

$$= \frac{1}{(-4)^2} = \boxed{\frac{1}{16}}$$

#11 $\left(\frac{x^{7/2}}{x^{2/3}}\right)^{-6}$

Karmen $= \dfrac{x^{\frac{7}{\cancel{2}}\cdot\frac{-\cancel{6}\,3}{1}}}{x^{\frac{2}{\cancel{3}}\cdot\frac{-\cancel{6}\,2}{1}}} = \dfrac{x^{-21}}{x^{-4}}$

$x^{-21-{-4}} = x^{-17} =$

$\dfrac{x^4}{x^{21}} = x^{-17} = \boxed{\frac{1}{x^{17}}}$

Ran $\left(\frac{x^{7/2}}{x^{2/3}}\right)^{-6} = \left(x^{7/2-2/3}\right)^{-6}$ $= \boxed{\frac{1}{x^{17}}}$

$$= \left(x^{\frac{21-4}{6}}\right)^{-6} = \left(x^{\frac{17}{\cancel{6}}}\right)^{-\cancel{6}} = x^{-17} \;//$$

#23 $\left(\frac{x^{15}}{y^{10}}\right)^{3/5} = \frac{x^{\frac{\cancel{15}^{\,3}}{1}\cdot\frac{3}{\cancel{5}}}}{y^{\frac{\cancel{10}^{\,2}}{1}\cdot\frac{3}{\cancel{5}}}} = \frac{x^9}{y^6}$

#13 $\frac{125^{4/3}}{125^{2/3}} = 125^{4/3-2/3} = 125^{2/3}$

$= \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

#53 $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

~~odd~~

$(-8)^{1/3} = -2$, $(8)^{1/3} = 2$

$(-16)^{1/4} =$ done $\quad (16)^{1/4} = 2$

$+ = (-2)\underbrace{(-2)(-2)}\underbrace{(2)}$

$+ = (2)(2)(2)(2)$

Fact

- Even roots are possible for positive number and zero.
- Odd roots are possible for ~~[illegible]~~ positive, negative, zero.