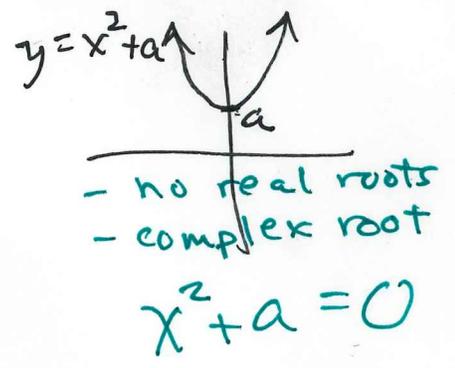
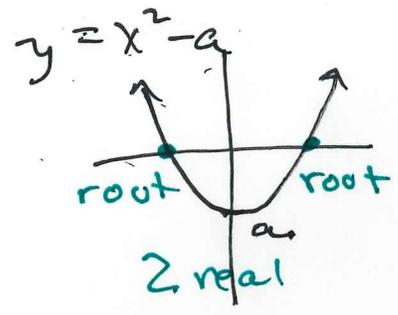
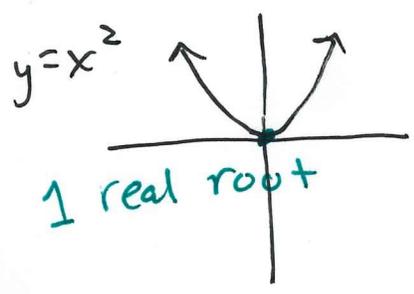


math 223

Aug 24

$\sqrt{-1} = (i)$

~~zings~~ imaginary #



$x^2 + a = 0$

$\sqrt{x^2} = \sqrt{-a}$

$x = \pm \sqrt{-a}$
no soln in reals

Set of Real numbers \mathbb{R} "a"



Build \mathbb{R} from its subsets

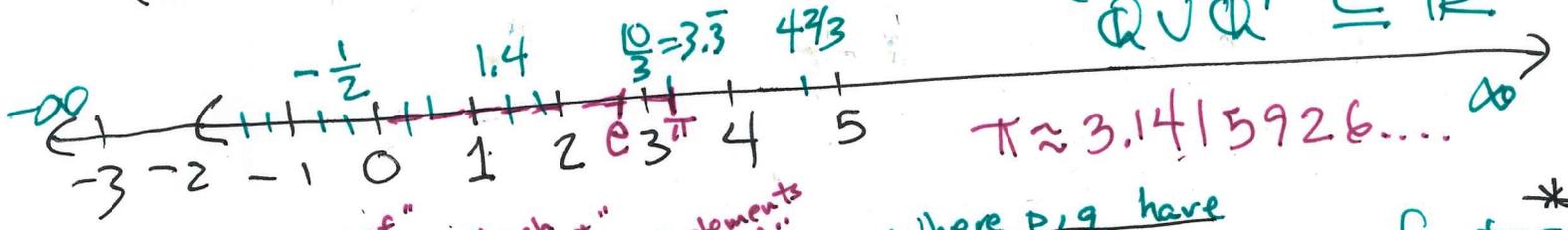
\mathbb{N} natural (counting)
 $\{1, 2, 3, \dots\}$

\mathbb{W} whole
 $\{0, 1, 2, \dots\}$

\mathbb{Z} integers
 $\{\dots, -3, -2, -1, 0, 1, \dots\}$

decimals
fractions
rational \mathbb{Q}
irrational \mathbb{Q}'
(not rational)
 $\mathbb{Q} \cup \mathbb{Q}' \subseteq \mathbb{R}$

reals \mathbb{R}
disjoint



** "set of" $\{ p/q \mid p, q \in \mathbb{Z}, \text{ where } p, q \text{ have no common factors} \}$
"such that"
"elements of"

$\boxed{\frac{4}{7}}$ =
$$\begin{array}{r} 0.57142 \\ 7 \overline{) 4.00000} \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 - 14 = 60 \end{array}$$

$\frac{4}{7} = 0.\overline{571428}$

* lowest terms

$20 - 14 = 60$

Union

$A \cup B$

elements in A

$a \in A$

or
elements in B

"a is an element of set A"

Domain of function

$x \in \mathbb{R}$
algebraic notation

$(-\infty, \infty)$
interval notation

Intersection

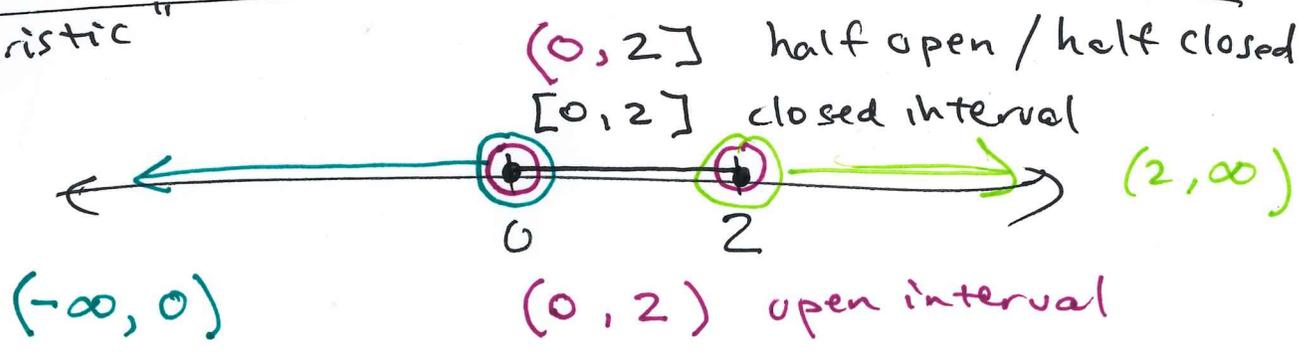
$A \cap B$

elements in A and B
at the same time

Rationals - "Countably infinite"
label ~~as~~

Irrationals - "Uncountably infinite"

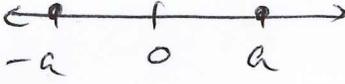
"heuristic"



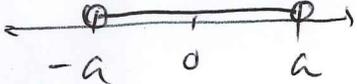
∞ - idea of unboundedness; not a number

Here are the items I didn't get to!

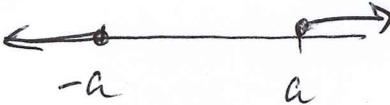
App. A - Absolute Value in One and Two Dimensions

Def $|x| = a$  $x = a, -a$

All x that lie a units from zero.

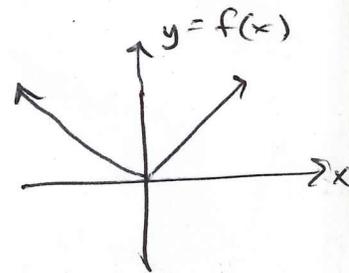
$|x| < a$  $-a < x < a, (-a, a)$

All x that lie less than a units from zero.

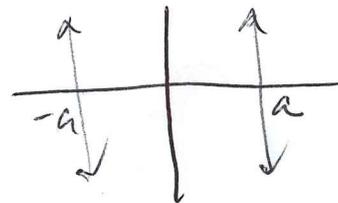
$|x| \geq a$  $x \geq a$ or $x \leq -a$
 $(-\infty, -a] \cup [a, \infty)$

All x that lie a units and beyond a units from zero

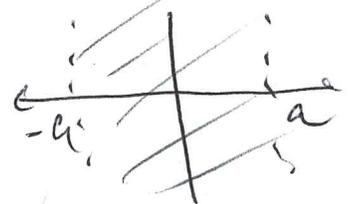
Def $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



$|x| = a$ is the graph of the vertical lines $x = a$ and $x = -a$



$|x| < a$ is the region of the graph between the vertical lines $x = -a$ and $x = a$



Since " $| \cdot |$ " indicates positive quantities must result from substituting appropriate x into an expression, then we can rewrite an abs value expression w/o the " $| \cdot |$ " as follows:

First, consider constants only:

Ex $|4 - 4.3| = 4.3 - 4$, since $4 - 4.3 < 0$, so removing the abs value fences requires we negate $4 - 4.3$; that is $-(4 - 4.3) = 4.3 - 4$.

The answer is of course 0.3.

Ex $|3.2 - \pi| = 3.2 - \pi$, since $3.2 - \pi > 0$ already. But $|\pi - 3.2|$ is also $3.2 - \pi$, since $\pi - 3.2 < 0$, so removing " $| \cdot |$ " means negating $\pi - 3.2$, or $-(\pi - 3.2) = 3.2 - \pi$.

Now consider expressions with x , and determine how it would be represented w/o the abs value fences that



Aug 26

Friday

11

Properties / facts of absolute value:

① $|a||b| = |ab|$

② $|a^2| = |a|^2$ or $|a^n| = |a|^n$

both from fact $\sqrt{a^2} = |a|$ for $a \in \mathbb{R}$

④ $|a| + |b| \geq |a+b|$

triangle inequality

ex $a = -2, b = 7$

$|a+b| = |-2+7| = 5$

$|a| + |b| = 2+7 = 9$

Proof (start)

By def, $|a+b| = \begin{cases} a+b, & a+b \geq 0 \\ -(a+b), & a+b < 0 \\ -a-b \end{cases}$

Since $a \leq |a|$ $b \leq |b|$
 $-a \leq |a|$ $-b \leq |b|$

sneaky observation - see video

then $a+b \leq |a| + |b|$
 $-(a+b) = -a + (-b) \leq |a| + (-b) \leq |a| + |b|$
 $|a+b| \leq |a| + |b|$ \square

$$|\text{expression in variable } x| = \begin{cases} \text{expression where it's } > 0 \\ -\text{expression where it's } < 0 \end{cases}$$

Ex $|x+3| = \begin{cases} x+3, & \cancel{x+3} > 0, x > -3 \\ -(x+3), & x+3 < 0, x < -3 \end{cases}$

$[-3, \infty)$
 $(-\infty, -3)$

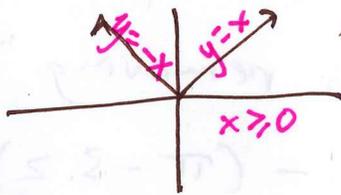
Illustrate the result with $x = -2$, ~~4~~

so $|-2+3| = |1| = 1$ ✓

and try $x = -4$, so $|-4+3| = |-1| = 1$ ✓



Important illustration is $f(x) = |x|$; $y = |x|$



$$y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \quad \text{tradition}$$

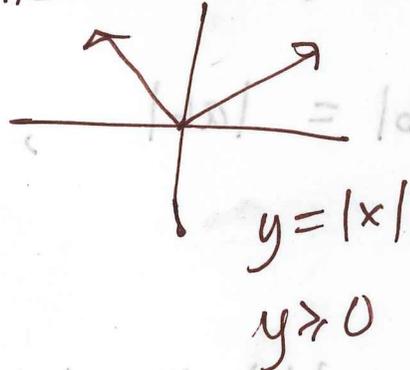
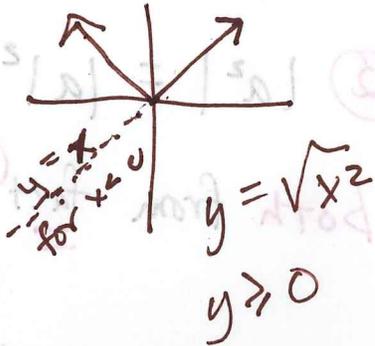
1)

(1a)

$$y = \sqrt{x^2} = x$$

for all $x \in \mathbb{R}$

$$y = |x|$$



$|a| = \sqrt{a^2}$
 $|a| = \sqrt{a^2}$
 for $a \in \mathbb{R}$

triangle inequality
 $|a+b| \leq |a| + |b|$
 $|a+b| = |-5+5| = 0$
 $|a| + |b| = 5+5 = 10$

$|a| + |b| \geq |a+b|$
 ex $a = -5, b = 5$

Proof (start)

$\begin{cases} a+b > 0 \\ a+b < 0 \end{cases}$

By def, $|a+b| =$

$a \geq |a|$
 $-a \geq |a|$
 $p \geq |p|$
 $-p \geq |p|$

$a+b \geq |a| + |b|$
 $-(a+b) = -a - b = (-a) + (-b) \geq |a| + |b|$

$|a+b| \geq (-a) + (-b) = |a| + |b|$
 $|a+b| \geq |a| + |b|$

reciprocal operation - see video

Process

(3)

Remove $| |$ from an expression in x

by stating values of x that produce a positive expression

Ex $|x-2| = \begin{cases} x-2 & \text{if } x \geq 2 \\ 2-x & \text{if } x < 2 \end{cases}$ (by convention, include = with top expression)

Ex $|3x+7| = \begin{cases} 3x+7 & \text{if } x \geq -7/3 \\ -(3x+7) & \text{if } x < -7/3 \end{cases}$

The result comes from the fact that an absolute value expression (say, that of a fn $f(x)$) is rewritten as two expressions without $| |$ for two "halves" of the number line.

Again, $|f(x)| = \begin{cases} f(x) & \text{where } f(x) \geq 0 \\ -f(x) & \text{where } f(x) < 0 \end{cases}$ } solve for x to get inter-
vals

Ex $f(x) = |x^3| = \begin{cases} x^3 & \text{where } x \geq 0 \\ -(x^3) & \text{where } x < 0 \end{cases}$

Illustrate: $x = -1$
 $x^3 = -1$, so $-(x^3) = +1$

Notice on this example, if $x < 0$ then $x^3 < 0$, so negating it results in a nonnegative value for any $x < 0$.

Properties of $|a|$:

- i) $|a^n| = |a|^n$
- ii) $|ab| = |a||b|$
- iii) $|\frac{a}{b}| = \frac{|a|}{|b|}$

$\sqrt{a^2} = |a|$ is true for all $a \in \mathbb{R}$. This is "obvious" in that squaring any no results in a positive no, whose sq. root is then positive. More on this later when we look at $f(x) = \sqrt{x^2}$ and $f(x) = |x|$

Prove $|ab| = |a||b|$ using $|a| = \sqrt{a^2}$

Proof $|ab| = \sqrt{(ab)^2} = \sqrt{a^2 b^2} = |a||b|$

Prove $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

Proof $\left|\frac{a}{b}\right| = \sqrt{\left(\frac{a}{b}\right)^2} = \sqrt{\frac{a^2}{b^2}} = \frac{\sqrt{a^2}}{\sqrt{b^2}} = \frac{|a|}{|b|}$

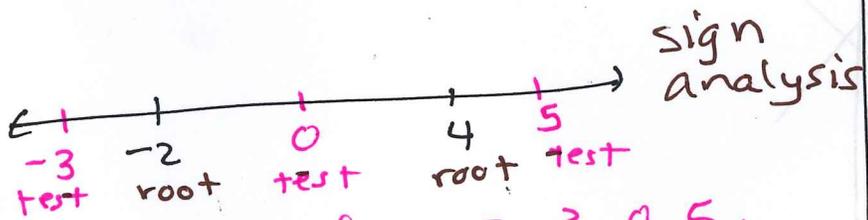
More homework from App A

#28 $x^2 < 2x + 8$

$y = x^2 - 2x - 8 < 0$ • first set terms opposite $<$
 $(x-4)(x+2) < 0$ • factor

The roots of $(x-4)(x+2) = 0$ • roots are $x = 4, -2$. Plot • # line

the roots on a # line and • Test test one value in each interval in the factored inequality.



Sign analysis for $x = -3, 0, 5$:

$x = -3, (-3-4)(-3+2) = +$

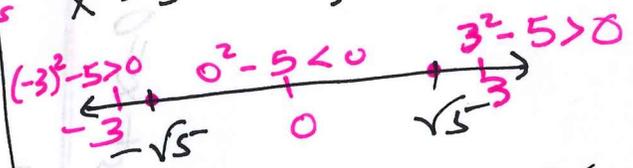
so reject the interval $(-\infty, -2)$

Sign analysis for $x = 0: (0-4)(0+2) = -$

so include $(-2, 4)$. Etc. Soln: $(-2, 4)$

#32 $x^2 \geq 5$

$x^2 - 5 \geq 0$; roots $x = \pm\sqrt{5}$



Soln: $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$

IMPORTANT PROPERTY

If $AB = 0$ then either $A = 0$ or $B = 0$.

There is no property that says "If $AB = C$ then either $A = C$ or $B = C$ ". Unless $C = 0$.

So turn any eqn or inequality

$AB = C$ into $AB - C = 0$ to solve

Prove

Proof $|ap| = \sqrt{(ap)}$

Proof $\frac{|a|}{|b|} = \frac{|a|}{|b|}$

Proof

$\sqrt{\left(\frac{a}{b}\right)^2}$

$|ap| = |a||p|$ remind $|a| = \sqrt{a^2}$

$\sqrt{a^2} \sqrt{p^2} = |a||p|$

strict inequality

$(x-4)(x+2) \oplus \oplus = + > 0$

test

More homework from App A

28

$x^2 - 2x - 8 < 0$
 $x^2 + 2x + 8 > 0$

sign:

$0 > (-) = (+) < 0$

poor

$(x-4)(x+2)$
+ check

in

test $x=0$

-2

IMPORTANT PROPERTY

IF $AB = 0$ THEN either $A = 0$ or $B = 0$

There is no property that says "If $AB = C$ then either $A = C$ or $B = C$ ". Unless $C = 0$.

to turn and eqn or inequality into $AB = C$ solve $AB - C = 0$ solve

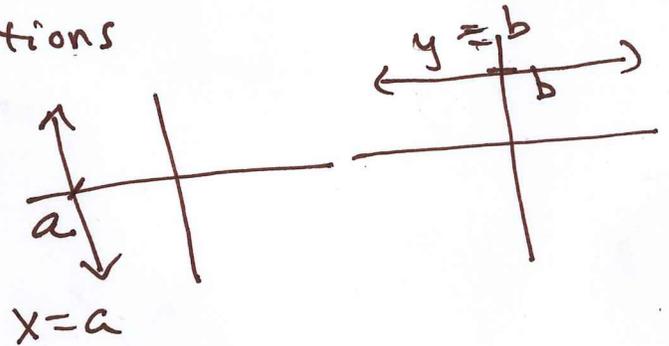
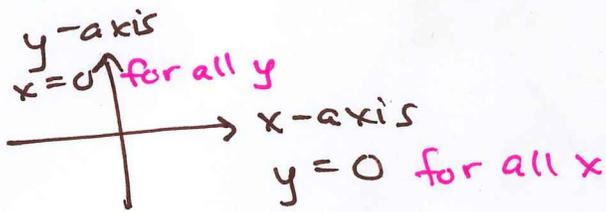
to include $(-5, 4)$: For $x = 0$: $(0-4)(0+5) = -20$
to reject the interval $(-\infty, -5)$:
 $x = -3$: $(-3-4)(-3+5) = +7$
Sign analysis for $x = -3, 0, 2$:
-3 -5
-3 -5
+ - +
root root not root

in the factored inequality: test one value in each interval the roots on a # line and one $x = 4, -5$. The roots of $(x-4)(x+5)$ are $x^2 - 2x - 8 < 0$ and $x^2 + 2x + 8 > 0$

test $x=0$
 $x > 2$
 $x < -2$
roots $x = \pm \sqrt{2}$

Lines, Properties, Facts

Def Set of pts through any 2 pts in "Locus" the x,y-plane and extending infinitely in both directions



Forms of linear eqns:

General: $Ax + By = C$

Slope-intercept: $y = mx + b$

m is slope, b is y-int. $(0, b)$

$m = \frac{\Delta y}{\Delta x}$

