

L^AT_EX submissions are mandatory. The template for this problem can be found on the Piazza resource page for this course.

Problem 1

Example 3.6 in Trefethen-Bau shows that if A is an outer product of two vectors $A = uv^t$, then $\|A\|_2 = \|u\|_2\|v\|_2$, where $\|\cdot\|_2$ denotes both the 2-norm on vectors (the usual Euclidean norm) and the corresponding induced operator norm on matrices.

Is the same true for the Frobenius norm, that is, is $\|A\|_F = \|u\|_F\|v\|_F$? Prove it or give a counterexample.

Solution:

Problem 2

Determine the reduced SVDs of the following matrices:

$$(a) \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}, (b) \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, (c) \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, (d) \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, (e) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

(Note that the answers can be different up to some multiplication of columns of U and V by ± 1 .)

Solution:

Problem 3

Determine the (reduced) SVD of the following matrix (by hand calculation):

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

Solution:

Problem 4

Suppose A is an $m \times n$ matrix and B is the $n \times m$ matrix obtained by rotating A ninety degrees clockwise on paper. Do A and B have the same singular values? Prove that the answer is yes or give a counterexample.

Solution: