LaTeXsubmissions are mandatory. The template for this problem can be found on the Piazza resource page for this course.

Problem 1

a. If $A^2 = I$, what are possible eigenvalues of A?

b. If this A is 2×2 and not I or $-I$, find its trace and determinant.

c. If the first row of this matrix is $(3, -1)$, what is the second row?

Solution:

Problem 2

(a) A 2×2 matrix A satisfies $\mathrm{tr}(A^2) = 5$ and $\mathrm{tr}(A) = 3$ (where $\mathrm{tr}(X)$ denotes the trace of X). Find $\det(A)$.

(b) A 2×2 matrix A has two proportional columns and $\mathrm{tr}(A) = 5$. Find $\mathrm{tr}(A^2)$.

(c) A 2×2 matrix A has $\det(A) = 5$ and positive integer eigenvalues. What is the trace of A?

Solution:

Problem 3

For each of the following statements, prove that it is true or give an example to show it is false. Throughout, A is a complex $m \times m$ matrix unless otherwise indicated.

a. If λ is an eigenvalue of A and $\mu \in \mathbb{C}$, then $\lambda - \mu$ is an eigenvalue of $A - \mu I$.

b. If A is real and λ is an eigenvalue of A, then so is $-\lambda$.

c. If A is real and λ is an eigenvalue of A, then so is $\overline{\lambda}$.

d. If λ is an eigenvalue of A and A is non-singular, then λ^{-1} is an eigenvalue of A^{-1}.

e. If all the eigenvalues of A are zero, then $A = 0$.

f. If A is diagonalizable and all its eigenvalues are equal, then A is diagonal.

g. If A is invertible and diagonalizable, then A^{-1} is diagonalizable.

h. Matrices A and A^t have the same eigenvalues.

Solution:

Problem 4

Suppose each "Gibonacci" number G_{k+2} is the average of the two previous numbers G_{k+1} and G_k. Then $G_{k+2} = \frac{1}{2}(G_{k+1} + G_k)$. In matrix form this can be written as

$$\begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix} = A \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}.$$

a. Find the eigenvalues and eigenvectors of A.

b. Find the limit of the matrices A^n as $n \to \infty$.

c. If $G_0 = 0$ and $G_1 = 1$, which number do the Gibonacci numbers approach?

Solution: