Math 447 - February 8, 2013 - Test 1

Name:______________________________

Read these instructions carefully: The points assigned are *not* meant to be a guide to the difficulty of the problems. If the question is multiple choice, there is a penalty for wrong answers, so that your expected score from guessing at random is zero. No partial credit is possible on multiple-choice and other no-work-required questions.

1. (25 points) Of the travelers arriving at a small airport, 60% fly on major airlines, 30% fly on privately owned planes, and the remainder fly on commercially owned planes not belonging to a major airline. Of those traveling on major airlines, 50% are traveling for business reasons, whereas 60% of those arriving on private planes and 90% of those arriving on other commercially owned planes are traveling for business reasons.

Suppose that we randomly select one person arriving at this airport, and wish to find the probability that this person arrived on a privately owned plane, given that the person is traveling for business reasons.

a. (5 points) Name the events defined in the problem. Hint: "Let M be the event that ..."

b. (10 points) State the equation that will allow you to obtain the desired probability, in terms of the notation established in part (a).

c. (5 points) The numbers in the problem give various probabilities. Write down these probabilities using the notation you established in part (a).

d. (5 points) Now solve the problem: find the probability that this person arrived on a privately owned plane, given that the person is traveling for business reasons. *To obtain credit your final answer must be expressed as a percentage—complex fractions are not acceptable!*

2. (20 points) A diagnostic test for a disease is such that it (correctly) detects the disease in 90% of the individuals who actually have the disease. Also, if a person does not have the disease, the test will report that he or she does not have it with probability .9. Only 1% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic test indicates that she has the disease, what is the conditional probability that she does, in fact, have the disease? *To obtain credit your final answer must*

be expressed as a percentage—complex fractions are not acceptable!

3. (40 points) (Monty Hall) In this version of the game you are shown 6 cards face-down, of which two are red queens, and four are black spot cards. The objective is to select a queen. After you make your initial selection, the host picks out another card (not your selection), and turns it over to show you it is black. (He can always do this because he knows where the red queens are.) He then offers you the opportunity to change your choice to one of the four remaining cards. (The ones which are neither your original selection nor the black card he showed you.)

a. (5 points) Establish notation: name the relevant events.

b. (5 points) What is the probability of winning by sticking with your original choice?

c. (15 points) What is the probability of winning by switching your choice?

d. (15 points) The prize for picking out a queen is \$100. After showing you the black card, but before you have made the choice to switch or to stay with your original choice, the host makes an offer: For \$15, you can see another black card before deciding whether to switch. Should you accept this offer and pay \$15?

4. (9 points) The release of two out of three prisoners has been announced. but their identity is kept secret. One of the prisoners considers asking a friendly guard to tell him who

is the prisoner other than himself that will be released, but hesitates based on the following rationale: at the prisoner's present state of knowledge, the probability of being released is 2/3, but after he knows the answer, the probability of being released will become 1/2, since there will be two prisoners (including himself) whose fate is unknown and exactly one of the two will be released.

Which of the following statements best describes the prisoner's reasoning and conclusion about the probability (p) of release after inquiring with the guard?

(a) The conclusion $p = 1/2$ is correct, but the reasoning is incorrect.

(b) The conclusion $p = 1/2$ is incorrect, and the reasoning is also incorrect.

(c) The conclusion $p = 1/2$ is correct, and the reasoning is correct also.

(d) The reasoning is partly right, and a corrected version of it shows that $p = 1/3$.

5. (16 points) Let A and B be independent events with $P(A) = 0.4$ and $P(B) = 0.25$

a. (6 points) What is the definition of independence? (Give the equation that must be satisfied for A and B to be independent.)

b. (5 points) Find $P(A \cup B)$

c. (5 points) Find $P(\bar{A} \cap \bar{B})$

6. (15 points) Vlad is to play a 2-game chess match with Gary and wishes to maximize his chances of winning, and minimize Gary's chances of winning. To do this, he may select a strategy right before he plays each game: timid or bold.

Unfortunately, Gary is the superior player. If Vlad plays timidly, Gary will still win 10 percent of those games, and the rest will be draws. If Vlad plays boldly, Gary will win 5/9 of those games, and lose the rest.

a. (6 points) Describe Vlad's optimal strategy in this 2-game match.

b. (9 points) Which of the following best describes the possible outcomes of a match between Vlad and Gary? (Assume Vlad uses his optimal strategy for the match in question.)

(a) Vlad is more likely to win the 2-game match than Gary. He would also be a favorite in a 50-game match, because he can vary his style of play.

(b) Gary is the superior player, and so is more likely to win the 2-game match, and also more likely to win a 50-game match.

(c) Gary is the favorite in the 2-game match, but in the long run Vlad's ability to vary his style of play will give him the advantage. Vlad is the favorite in a 50-game match.

(d) Vlad is more likely to win the 2-game match than Gary, but is an underdog in a 50-game match.

6. (15 points) Suppose that there is a 1 in 10 chance of injury on a single skydiving attempt.
a. (6 points) If we assume that the outcomes of different jumps are independent, what is the probability that a skydiver is injured if she jumps twice?

b. (9 points) A friend claims if there is a 1 in 10 chance of injury on a single jump then there is a 100% chance of injury if a skydiver jumps 10 times. Is your friend correct? What, approximately, is the probability of being injured in 10 jumps? *To obtain credit your final answer must be expressed as a percentage—complex fractions are not acceptable!*