Math 447 - April 23, 2013 - Test 3

Name:_____

Read these instructions carefully: The points assigned are *not* meant to be a guide to the difficulty of the problems. If the question is multiple choice, there is a penalty for wrong answers, so that your expected score from guessing at random is zero. No partial credit is possible on multiple-choice and other no-work-required questions.

1. (6 points) A bag contains 99 normal (fair) coins and one gaffed coin which has two heads. A single coin is selected at random from the bag and flipped 6 times. It comes up heads every time. The chance that it is actually the two-headed coin is closest to:

(a) 0.20
(b) 0.40
(c) 0.60
(d) 0.80

2. (6 points) A woman has two children, one of whom is a boy born on a Tuesday. Select from (a)-(d) below the answer closest to the probability that both children are boys.

If you need the event and sample space specified more precisely to answer, here is a description of the process for selecting a random woman with two children and the attendant assumptions: We gather all those women in the world who have exactly two children, tell each of them to "go home unless you have a boy born on a Tuesday", and select a woman randomly from those who remain. Assume that births are equally likely to occur on any day of the week, and that on any given day, boys and girls are equally likely.

(a) 0.51
(b) 0.48
(c) 0.34
(d) 0.26

3. (6 points) Before going on vacation for a week, you ask your absent-minded friend to water your ailing plant. Without water, the plant has a 90 percent chance of dying. Even with proper watering, it has a 20 percent chance of dying. And the probability that your friend will forget to water it is 30 percent.

If your friend forgets to water the plant, the probability it will be dead when you return is closest to:

(a) 0.93
(b) 0.83
(c) 0.73
(d) 0.63

4. (6 points) (*(Exactly the same story and numbers as the previous problem, but a slightly different question.)* Before going on vacation for a week, you ask your absent-minded friend to water your ailing plant. Without water, the plant has a 90 percent chance of dying. Even with proper watering, it has a 20 percent chance of dying. And the probability that your friend will forget to water it is 30 percent.

If the plant is dead when you return, the probability that your friend forgot to water it is closest to:

(a) 0.62
(b) 0.72
(c) 0.82
(d) 0.92

5. (6 points) Scores on an examination are normally distributed with mean 78 and standard deviation 6. Which of the following numbers is closest to the probability that a student's score exceeds 84, given that it exceeds 72?

(a) 0.14
(b) 0.16
(c) 0.18
(d) 0.21

6. (6 points) Scores on an examination are normally distributed with mean 78 and standard deviation 6.

Which of the following numbers is closest to the proportion of students who have scores 5 or more points above the score that cuts off the lowest 25%?

(a) 0.45
(b) 0.53
(c) 0.60
(d) 0.68

7. (9 points) Suppose that a random variable Y has a probability density function given by

$$f(y) = \begin{cases} ky^3 e^{-y/2}, & y > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

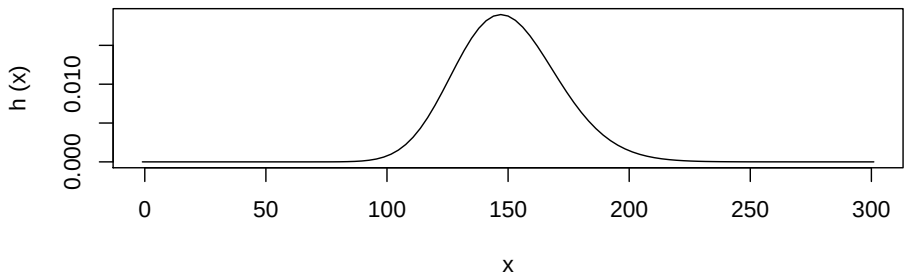
a. (3 points) The value of k that makes f a probability density function is closest to

(a) 0.01
(b) 0.5
(c) 1
(d) 12

b. (6 points) The value of $E[Y^2]$ is closest to

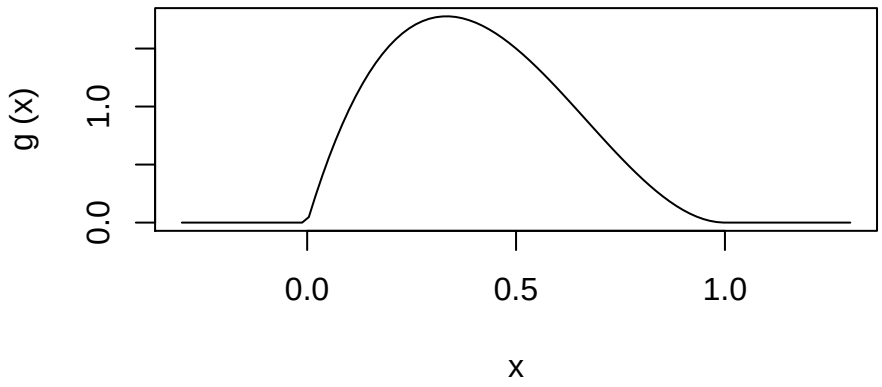
(a) 63
(b) 101
(c) 109
(d) 120

8. (12 points) Let Z be a standard normal random variable. Find, with proof, $E[Z^4]$.



9. (4 points) From which of the following distributions does the probability density function graphed above arise?

(b) the beta distribution with parameters $\alpha = 1$ and $\beta = 2$
(b) the normal distribution with parameters $\mu = 150$ and $\sigma = 5$
(c) the gamma distribution with parameters $\alpha = 50$ and $\beta = 3$
(d) the chi-square distribution with 100 degrees of freedom
(e) none of the above



10. (4 points) From which of the following distributions does the probability density function graphed above arise?

(a) the beta distribution with parameters $\alpha = 3$ and $\beta = 3$

(b) the beta distribution with parameters $\alpha = 5$ and $\beta = 5$

(c) the normal distribution with parameters $\mu = 0.3$ and $\sigma = 0.5$

(d) the beta distribution with parameters $\alpha = 2$ and $\beta = 3$

(e) none of the above

11. (32 points) Let X and Y be random variables representing the coordinates of a point which is chosen at random from the triangle in the (x, y) plane with vertices $(-1, 0)$, $(1, 0)$, and $(0, 1)$.

a. (4 points) Write down a formula for the probability density function $f(x, y)$.

b. (4 points) Give the *definition* of the marginal density function $f_X(x)$.

c. (6 points) Find $f_X(x)$. Splitting into cases if necessary, be sure your formula is valid for all real values of x.

d. (4 points) Give the *definition* of the conditional density function $f(y \mid X = x)$.

e. (4 points) Give the *definition* of $\mathrm{Cov}(X, Y)$.

f. (10 points) Find $\mathrm{Cov}(X, Y)$.

g. (4 points) Are X and Y independent? (Only an answer is necessary, no reasoning required.)

12. (6 points) Suppose the moment generating function of Y is $1/(1-3t)$. What is the distribution of Y? Indicate the values of the parameters associated with this distribution.

13. (10 points) Suppose that the random variables X and Y are such that $E[X] = 4$, $E[Y] = -1$, $V[X] = 2$, and $V[Y] = 8$.
a. (2 points) Find $\text{Cov}(X, X)$.

b. (8 points) What is the largest possible value of $\text{Cov}(X, Y)$?

14. (12 points) Suppose that W is normal with mean 3 and variance 1. Suppose X has the gamma distribution with parameters $\alpha = 2$ and $\beta = 1$. Let $Y = 2W - 1$ and $Z = 2X - 1$.
a. (6 points) Which of the following statements about the random variables Y and Z is most accurate?

(a) Y is normally distributed and Z has the gamma distribution.
(b) Y is not normally distributed but Z has the gamma distribution.
(c) Y is not normally distributed and Z does not have the gamma distribution.
(d) Y is normally distributed but Z does not have the gamma distribution.

b. (6 points) Prove that your answer in part (a) is correct.