Math 447 **Test 1** **Fall 2015**

No books, no notes, only SOA-approved calculators. You must show work, unless the question is a true/false or fill-in-the-blank question.

Name: ______________________________ Section: ____________________

Question	Points	Score
1	10	
2	10	
3	12	
4	6	
5	15	
6	12	
7	5	
Total:	70	

1. Let Y has distribution function

$$F(y) = \begin{cases} 0, & y0, \\ \frac{y}{8}, & 0 < y < 2, \\ \frac{y^2}{16}, & 2 \leq y < 4, \\ 1, & y \geq 4. \end{cases}$$

(a) (3 points) What is the density of Y?

(b) (3 points) Find the mean of Y.

(c) (4 points) Find the variance of Y.

2. Scores on an examination are assumed to be normally distributed with mean 78 and variance 36.

   (a) (5 points) Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?

   (b) (5 points) If it is known that a student's score exceeds 72, what is the probability that his or her score exceeds 84?

3. (6 points) Let Y have a probability density function given by

$$f(y) = \begin{cases} 4y^2 e^{2y}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) (3 points) Find $E[Y]$

   (b) (3 points) Find $V[Y]$

4. Let Y be an exponential r.v. with parameter $\beta = 1$.

(a) (3 points) Find $P(|Y - \mu| \geq 2\sigma)$, where μ and σ are the expectation and standard deviation of Y.

(b) (3 points) Compare with the corresponding probabilistic estimate given by Chebyshev's theorem.

5. The joint density function of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 30y_1y_2^2, & y_1 - 1 \leq y_2 \leq 1 - y_1, 0 \leq y_1 \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) (3 points) Find $P(Y_1 > Y_2)$.

(b) (3 points) Derive the marginal density of Y_1.

(c) (3 points) Derive the conditional density of Y_2 given $Y_1 = y_1$.

(d) (3 points) Find $P(Y_2 > 0 | Y_1 = .75)$.

(e) (3 points) Are the random variables Y_1 and Y_2 independent?

6. Suppose that the probability that a head appears when a coin is tossed is p and the probability that a tail occurs is $q = 1 - p$. Person A tosses the coin until the first head appears and stops. Person B does likewise. Let Y_1 and Y_2 denote the number of times that persons A and B toss the coin, respectively. It is reasonable to assume that Y_1 and Y_2 are independent.

   (a) (3 points) What is the probability that A and B stop on exactly the same number toss?

   (b) (3 points) Find $E(Y_1)$

   (c) (3 points) Find $V(Y_1)$

   (d) (3 points) Find $V(Y_1 - Y_2)$

7. (5 points) Let Y_1 have an exponential distribution with mean λ and the conditional density of Y_2 given $Y_1 = y_1$ be

$$f(y_2|y_1) = \begin{cases} \frac{1}{y_1}, & 0 \le y_2 \le y_1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find $E(Y_2)$, the unconditional mean of Y_2.