

Q1 Let X and Y have the joint probability density function given by

$$f(x, y) = 2, \text{ if } 0 \leq x \leq 1, 0 \leq y \leq 1, x + y \leq 1, \text{ and } 0 \text{ otherwise.}$$

Find $\mathbb{P}(X > 1/8 | Y > 1/2)$.

- (A) 9/128 (B) 9/64 (C) 3/4 (D) 9/16 (E) other

Q2 Suppose X and Y are r.v.'s with joint density

$$f(x, y) = 1 \text{ for } 0 < x < 1, 0 < y < 1, \text{ and } 0 \text{ elsewhere.}$$

Find the density $f_U(u)$ of $U = XY$ on $[0, 1]$.

- (A) $2u$ (B) $-\log u$ (C) 1 (D) $u - u \log u$ (E) other

Q3 If $X \sim N(0, 1)$, what is the density of X^3 ?

- (A) $ce^{-\frac{1}{2}y^{2/3}}$ (B) $cy^2e^{-\frac{1}{2}y^{2/3}}$ (C) $cy^{-2/3}e^{-\frac{1}{2}y^{2/3}}$
 (D) $cy^{2/3}e^{-\frac{1}{2}y^{2/3}}$ (E) other

Q4 Polina and Anton are often late to pick up their daughter from daycare. The daycare charges \$1 per minute for tardiness. Suppose that the amount of time that they are late each day is distributed as $\text{Exp}(6)$. (So mean is 6 and std is 6.) Their child will be in daycare for 100 days this year. What is the probability that they will pay more than \$630?

- (A) $\mathbb{P}(Z > 0.5)$ (B) $\mathbb{P}(Z > 5)$ (C) $\mathbb{P}(Z > 0.05)$ (D) $\mathbb{P}(Z > 2)$ (E) other