Homework 9 MATH 304 Section 3 Solution

Assigned: Friday, October 10.
Potentially Collected: Friday, October 17.

1. Are the following subspaces of their respective vector spaces?

   (i) H consists of all vectors $\begin{bmatrix} s \\ 3s \\ 2s \end{bmatrix}$ in $\mathbb{R}^3$.

   (ii) W consists of all vectors $\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix}$ in $\mathbb{R}^4$.

   (iii) T is the set of all vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ where $x^2+y^2 \leq 1$ in $\mathbb{R}^2$.

(i) H is the span of $\left\{\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}\right\}$ as every element is of the form $\left\{ s\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \,\middle|\, s \in \mathbb{R} \right\}$

So H is a subspace $\mathbb{R}^3$.

(ii) W is the span of $\left\{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix}\right\}$ as every element is of the form $\left\{ s\begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix} \,\middle|\, s,t \in \mathbb{R} \right\}$.

So W is a subspace $\mathbb{R}^4$.

(iii) T is not a subspace as $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \in T$ (note $1^2+0^2 \leq 1$) yet $c\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c \\ 0 \end{bmatrix}$ is generally not in T.

If $c>1$, $c^2+0^2=c^2>1$

so $c\begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin T$.

2. Let $\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

   (i) Is $\vec{w}$ in Span$\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$?

   (ii) If yes, then what are the coefficients for $\vec{w}$ in the linear combination of $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$?

Solve $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{array}\right] \xrightarrow{r_3=R_3+R_1} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 5 & 10 & 5 \end{array}\right] \xrightarrow{r_3=R_3-5R_2} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$r_1=R_1-2R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ so $\vec{w} = a_1\vec{v_1} + a_2\vec{v_2} + a_3\vec{v}$

where $a_1 = 1$

$a_2 = 1-2a_3$

a_3 anything

parametric solutions

$\begin{bmatrix} 1 \\ 1-2a_3 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + a_3\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$