

Homework 22 MATH 304 Section 3

Solution

Assigned: Wednesday, November 19.
Potentially Collected: Monday, December 1.

1. Find the values of t which are solutions to the following equations.

(a) $\det \begin{pmatrix} t-2 & 2 \\ 3 & t-3 \end{pmatrix} = 0$ $\xrightarrow{\quad} (t-2)(t-3) - 6 = 0 \Rightarrow t^2 - 5t + 6 - 6 = 0$
 $\Rightarrow t(t-5) = 0$ $\{t \in \{0, 5\}$

(b) $\det \begin{pmatrix} t-1 & -4 \\ 0 & t-4 \end{pmatrix} = 0$ $\xrightarrow{\quad} (t-1)(t-4) = 0$ $t \in \{1, 4\}$

(c) $\det \begin{pmatrix} t-1 & 0 & 1 \\ -2 & t+2 & -1 \\ 0 & 0 & t+1 \end{pmatrix} = 0$

2. Using determinants, which of the following matrices are singular?

(a) $\det \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 0 \end{pmatrix} = 1(-1)^{1+3} \det \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} = (1-2) = -1$
 $\xrightarrow{\quad} 3^{\text{rd}} \text{ column expansion}$

the matrix is invertible as its determinant is non-zero

(b) $\det \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ -3 & 1 & 2 \end{pmatrix} = 2(-1)^{2+1} \det \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} + 1(-1)^{2+2} \det \begin{pmatrix} 1 & 3 \\ -3 & 2 \end{pmatrix}$ $\xrightarrow{\quad} 2^{\text{nd}} \text{ row expansion}$
 $= -2(4-3) + (2+9) = -2 + 11 = 9$ Invertible

3. Suppose the matrix A satisfies the equation $A^2 = A$. Show that either A is singular or $\det(A) = 1$.

If $A^2 = A$ then $\det(A^2) = \det(A)$

$\Rightarrow \det(A)^2 - \det(A) = 0$

$\Rightarrow \det(A)(\det(A) - 1) = 0$

either $\det(A) = 0$ (which implies A is singular)
or $\det(A) = 1$.

(c) $0 = (t+1)(-1)^{3+3} \det \begin{pmatrix} t-1 & 0 \\ -2 & t+2 \end{pmatrix}$ $\xrightarrow{\quad} 3^{\text{rd}} \text{ row expansion}$
 $= (t+1)(t-1)(t+2)$ $t \in \{-1, 1, -2\}$