Homework 27 MATH 304 Section 3

Assigned: Monday, December 8.
Potentially Collected: Friday, December 12.

1. Find an orthonormal basis for the subspace of $\mathbb{R}^3$ spanned by

$$\mathcal{X} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} \right\}$$

$\text{Span}(\mathcal{X}) = \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}\right)$

2. Find an orthonormal basis for the null space of each of the following matrices

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 2 & -6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 2 \end{bmatrix}$$

Use the Gram-Schmidt Process

3. Find an orthonormal basis for each of the following subspaces.

(a) $\left\{ \begin{bmatrix} a \\ a+b \\ b \end{bmatrix} : a, b \in \mathbb{R} \right\}$

(b) $\left\{ \begin{bmatrix} a \\ a+b \\ c \\ b+c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$

(c) $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a+b+c=0 \right\}$

(d) $\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : a-b-2c+d=0 \right\}$

$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

$\vec{v}_2 = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} - \text{proj}_{\vec{v}_1}\left(\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}\right)$

$= \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} - \left(\frac{-3+0-2}{5}\right)\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

$= \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$

$(\vec{v}_1, \vec{v}_2)$ is an orthogonal basis for span($\mathcal{X}$).

$\left(\frac{\vec{v}_1}{\|\vec{v}_1\|}, \frac{\vec{v}_2}{\|\vec{v}_2\|}\right) = \left(\begin{bmatrix} 1/\sqrt{5} \\ 0 \\ -2/\sqrt{5} \end{bmatrix}, \begin{bmatrix} -2/3 \\ 2/3 \\ -1/3 \end{bmatrix}\right)$ is an orthonormal basis for Span($\mathcal{X}$).