Homework 26 MATH 304 Section 3

(b) $||\vec{u}-\vec{v}|| = ||\begin{bmatrix} -3 \\ 7 \end{bmatrix}|| = \sqrt{(-3)^2+7^2} = \sqrt{58}$

$||\vec{w}-\vec{v}|| = ||\begin{bmatrix} -8 \\ 0 \end{bmatrix}|| = 8$

Assigned: Wednesday, December 3.
Potentially Collected: Wednesday, December 10.

1. Let $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$, $\vec{a} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix}$, and $\vec{c} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$.
   (a) Find $||\vec{a}||$, $||\vec{b}||$, and $||\vec{c}||$.
   (b) Find $||\vec{u}-\vec{v}||$ and $||\vec{w}-\vec{v}||$.
   (c) Find $\text{proj}_{\vec{a}}(\vec{c})$ and $\text{proj}_{\vec{b}}(\vec{c})$.
   (d) Find $\text{proj}_{\vec{v}}(\vec{u})$ and $\text{proj}_{\vec{v}}(\vec{w})$.
   (e) Find $\vec{v}\cdot\vec{w}$ and $\vec{a}\cdot\vec{b}$.

(a) $||\vec{a}|| = \sqrt{0^2+(-2)^2+0^2} = 2$

$||\vec{b}|| = \sqrt{(-1)^2+(-3)^2+(-4)^2} = \sqrt{26}$

$||\vec{c}|| = \sqrt{1^2+(-2)^2+4^2} = \sqrt{21}$

2. Which of the following vectors are orthagonal? in the same direction? in opposing directions?

$$\vec{a} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad \vec{c} = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} \quad \vec{d} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \vec{u} = \begin{bmatrix} \frac{-1}{2} \\ 0 \\ \frac{-1}{4} \end{bmatrix}$$

3. Let $\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Find all the vectors $\vec{v}$ where $\vec{v}\perp\vec{w}$ and $\vec{v}\perp\vec{x}$.

(c) $\text{proj}_{\vec{a}}(\vec{c}) = \left(\frac{\vec{a}\cdot\vec{c}}{||\vec{a}||^2}\right)\vec{a} = \left(\frac{4}{4}\right)\begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$

$\text{proj}_{\vec{b}}(\vec{c}) = \left(\frac{\vec{c}\cdot\vec{b}}{||\vec{b}||^2}\right)\vec{b} = \left(\frac{-11}{26}\right)\begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix} = \begin{bmatrix} 11/26 \\ 33/26 \\ 44/26 \end{bmatrix}$

(d) $\vec{v}\cdot\vec{w} = 9$ $\qquad$ $\vec{a}\cdot\vec{b} = 6$

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(3) the vectors $\vec{v}$ orthogonal to both $\vec{w}$ and $\vec{x}$ form the orthogonal complement $\text{Span}(\vec{w},\vec{x})^\perp$.

$\text{Span}(\vec{w},\vec{x})^\perp = \text{nul}\left(\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}\right) = \text{span}\left(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right)$

(RREF $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$