True or **False**

If a piece of string has been chopped into n small pieces and the i^{th} piece is Δx_i inches long, then the total length of the string is exactly $\sum_{i=1}^{n} \Delta x_i$.

The table gives the values of a function obtained from an experiment. Use them to estimate $\int_3^9 f(x)dx$ using three equal subintervals:

- first with right endpoints,
- then with left endpoints,
- and finally with midpoints.

x	3	4	5	6	7	8	9
$f(x)$	-3.6	-2.1	-0.5	0.3	0.8	1.5	1.8

If the function is known to be an increasing function, can you say whether any of your three estimates is less than or greater than the exact value of the integral?

Express

$$\int_1^3 \frac{x}{2+x^4}\,dx$$

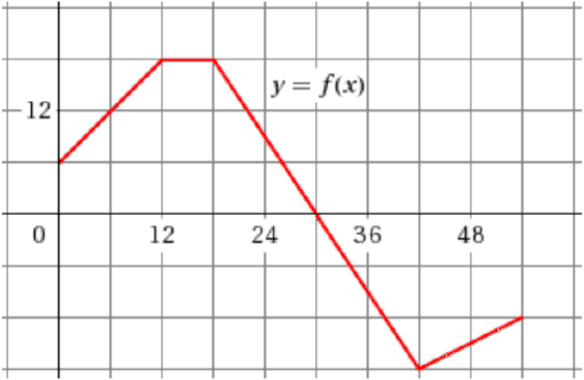
as a limit of Riemann sums.

Express

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos\left(2\pi+\frac{\pi}{n}\right)}{2\pi+\frac{\pi}{n}}\left(\frac{\pi}{n}\right)$$

as a definite integral.

The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



$$\int_0^{12} f(x)dx \qquad \int_0^{30} f(x)dx \qquad \int_{30}^{42} f(x)dx$$

Evaluate each integral by interpreting it in terms of areas.

a)

$$\int_{-10}^{0} \left(4 + \sqrt{100 - x^2}\right) dx$$

b)

$$\int_{0}^{12} |x - 6| dx$$

If $\int_0^6 f(x)dx = 9$ and $\int_0^6 g(x)dx = 4$, find $\int_0^6 (3f(x) + 4g(x))dx$

Given that $\int_0^1 15x\sqrt{x^2+4}dx = 25\sqrt{5} - 40$, what is

$$\int_1^0 15u\sqrt{u^2+4}du \text{ ?}$$

Given that $\int_0^1 x^2 dx = \frac{1}{3}$, what is $\int_0^1 (8 - 6x^2)dx$?

Given that $\int_a^b x dx = \frac{b^2 - a^2}{2}$ and $\int_0^{\pi/2} \cos(x) dx = 1$, evaluate

$$\int_0^{\pi/2} (2\cos(x) - 4x) dx$$

Write $\int_{-6}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-6}^{-3} f(x)dx$ as a single integral.

If $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 7$, find $\int_1^4 f(x)dx$.

Suppose f has absolute minimum value m and absolute maximum value M. What bounds can you give for $\int_3^6 f(x)dx$?

This problem and the next are a preview of Section 4.3.

a) Draw the graph $y = 2t + 1$ and use geometry to find the area under this line, above the t-axis, and between the vertical lines $t = 1$ and $t = 3$.

b) If $x > 1$, let $A(x)$ be the area of the region that lies under the line $y = 2t + 1$ between $t = 1$ and $t = x$. Sketch this region and use geometry to find an expression for $A(x)$.

c) Differentiate $A(x)$. Notice anything?

a) If $x > 1$, let

$$A(x) = \int_1^x (1 + t^2)dt.$$

$A(x)$ represents the area of a region. Sketch that region.

b) Given that $\int_a^b t^2 dt = \frac{b^3 - a^3}{3}$, find an expression for $A(x)$.

c) Differentiate $A(x)$. Notice anything?

d) If $x \geq 1$ and h is a small positive number, then $A(x + h) - A(x)$ represents the area of a region. Sketch that region.

e) Draw a rectangle that approximates the region from (d). Use this approximation to see that

$$\frac{A(x + h) - A(x)}{h} \approx 1 + x^2$$

f) Use part (e) to give an intuitive explanation for the result of part (c).