Quiz 5

April 5th, 2017

1) SHORT ANSWER: Briefly explain AND draw an example or a counterexample.

   a) If G is planar, and $e \in E(G)$, $G - e$ is planar.

   b) If G is planar, and $v \in V(G)$, $G - v$ is planar.

   c) Every subgraph of a planar graph is planar.

   d) Not every subgraph of a non-planar graph is non-planar.

   e) What is the contrapositive of the statement in part c)?

   f) If G is planar, and $e \in E(G)$, G/e is planar.

   g) What is the contrapositive of the statement in part f)?

   h) Is every graph (not necessarily simple) on at most 4 vertices planar?

   i) If G is simple and planar, every graph obtained from G by adding loops or parallel edges to it is planar.

   j) Euler's formula does not hold for disconnected graphs. Give the *smallest* nonempty planar simple graph for which Euler's formula does not hold.

2) Parts $a) - g)$ of question 1) motivate the following idea: a *minor* of a graph G is a graph that can be obtained from G by a sequence of deletions and contractions.

   a) Every subgraph of G is a minor of G, but not conversely. Draw a graph G and a minor of it that is not a subgraph of it.

   b) Is the simple graph on 5 vertices with 9 edges planar?

   c) Is the graph obtained from $K_{3,3}$ by deleting an edge planar?

   d) Why did I say "Is the" rather than "Are all" in parts b) and c)?

   e) $1h$), $2b$), and $2c$) together say that K_5 and $K_{3,3}$ are the smallest non-planar graphs. As it turns out, they are crucial:

      Kuratowski's Theorem: A graph is planar if and only if it does not have K_5 or $K_{3,3}$ as a minor.

      $1e$) and $1g$) prove the "easy" direction. State the "easy" direction and explain why those prove it.

rules:

1. Work out the problems on scratch paper, and transfer your neat and clean solutions onto the blue book provided. Turn in only the blue book (make sure all your names are on it).

2. The quiz is open book and open notes, but cell phones may not be out.

3. Failure of any group member to follow the rules will result in a score of zero for every group member.