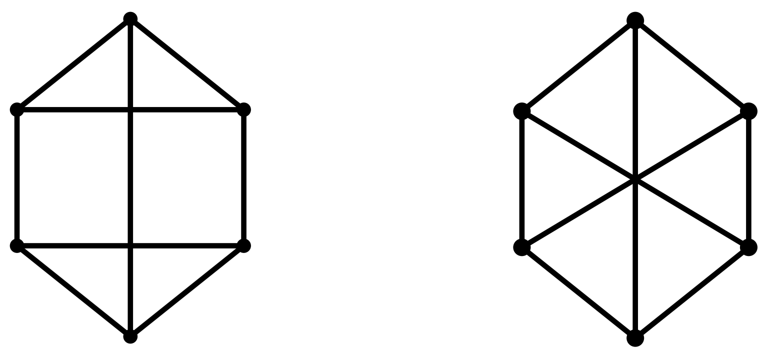
Math 314: Quiz 3 Questions

Questions:

(1)

A: The two graphs shown below have the same degree sequence, the same connectivity, they both have Hamilton cycles, neither has an Eulerian walk... lots of stuff. But they are not "essentially the same". Give a valid reason *other than the one from class* why not.



B: Prove that the complement of a regular graph is regular. If G is regular of degree k, what is the degree of the vertices in $\overline{G}$?

C: Given a graph G with n vertices, m edges, and c connected components, how many vertices does $\overline{G}$ have? How many edges? Give two graphs G_1 and G_2 with the same number of components such that $\overline{G}_1$ and $\overline{G}_2$ have different numbers of components.

(2)

A: Ten players participate in a chess tournament. Eleven games have already been played. Prove that there is a player who has played at least three games.

B: Let G be a connected regular graph with 22 edges. How many vertices can G have?

C: Let G be connected graph with at least 2 vertices. Prove there is a vertex that can be deleted so that the remaining subgraph is still connected.

(3)

A: Prove that any graph with a Hamilton cycle has connectivity at least 2.

B: Give a 2-connected graph without a Hamilton cycle.

C: We know the most edges a graph on n vertices can have is $\binom{n}{2}$. What is the smallest number of edges a *connected* graph on n vertices must have? (BONUS: What is the largest number of edges a *disconnected* graph on n vertices can have? (Can be done using calculus techniques, surprisingly))

(4)

A: Let G be graph with $\delta(G) = k$. Prove that G contains (as a subgraph) a path of length at least k.

B: Prove that if all vertices of G have even degree, then G has no cut edge.

C: Prove that if a graph G has *exactly* 2 vertices of odd degree, then there must be a path joining them.