Math 227 Sections 29 and 33 Quiz 10 5/4/18

Name:____________________________

1. What is the Taylor polynomial $T_2(x)$ centered at $a = 0$ for $\cos x$? Use it to estimate $\cos \frac{1}{2}$. You don't have to show any work for this problem.

$$T_2(x) = 1 - \frac{x^2}{2}$$

$$\cos\left(\tfrac{1}{2}\right) \approx T_2\left(\tfrac{1}{2}\right) = 1 - \frac{\left(\frac{1}{2}\right)^2}{2} = \frac{7}{8}$$

2. Estimate the accuracy of your estimate by using either Taylor's Inequality or the Alternating Series Estimation Theorem. State clearly which method you use.

Method 1: $\left|\frac{d^3}{dx^3}\cos(x)\right| \leq 1 = M$ for all $|x| \leq \frac{1}{2} = d$.

By Taylor's Inequality,

$$|R_2(x)| \leq \frac{M}{(2+1)!}|x|^{2+1} = \frac{1}{3!}|x|^3 \leq \frac{1}{3!}\left(\frac{1}{2}\right)^3 = \frac{1}{48} \text{ for } |x| \leq \frac{1}{2}$$

So $\frac{7}{8} - \frac{1}{48} \leq \cos\left(\frac{1}{2}\right) \leq \frac{7}{8} + \frac{1}{48}$

Method 2: $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$ for all $|x| < \infty$,

so $s = \cos\left(\frac{1}{2}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(\frac{1}{2}\right)^{2n}$. This is an alternating series with $b_n = \frac{1}{(2n)!}\left(\frac{1}{2}\right)^{2n}$. $T_2(x) = 1 - \frac{\left(\frac{1}{2}\right)}{2} = b_0 - b_1 = s_2$.

By the Alternating series Estimation Theorem,

$|s - s_1| \leq b_2 = \frac{1}{4!}\left(\frac{1}{2}\right)^4$. So $\frac{7}{8} - \frac{1}{4!\,2^4} \leq \cos\left(\frac{1}{2}\right) \leq \frac{7}{8} + \frac{1}{4!\,2^4}$.