**Math 227 Sections 29 and 33 Quiz 9** 4/27/18

**Name:**______________________________

Find the radius of convergence and the interval of convergence of the series.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$ Root test: $\displaystyle\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(x-2)^n}{n^n}\right|}$

$$= \lim_{n\to\infty} \sqrt[n]{\left(\frac{|x-2|}{n}\right)^n} = \lim_{n\to\infty} \frac{|x-2|}{n} = 0 < 1$$

$$\boxed{R = \infty}$$

$$\boxed{I = (-\infty, \infty)}$$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(x-1)^n$ Ratio test: $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$$= \lim_{n\to\infty} \left|\frac{(-1)^{n+1}(x-1)^{n+1}(2n-1)2^n}{(2n+1)2^{n+1}(-1)^n(x-1)^n}\right| = \lim_{n\to\infty} \frac{2n-1}{2(2n+1)}|x-1| = \frac{|x-1|}{2} < 1$$

$$|x-1| < \boxed{2 = R}$$

Test at $x=-1$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(-2)^n = \sum_{n=1}^{\infty} \frac{1}{2n-1}$ $\quad 0 \le \frac{1}{2n-1} \le \frac{1}{2n}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$ diverges $(p = 1 \le 1)$, so $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges by comparison.

Test at $x=3$: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(2)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$ $\quad b_n = \frac{1}{2n-1}$

$b_{n+1} \le b_n$, $\displaystyle\lim_{n\to\infty} b_n = 0$, so $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$ converges by the AST.

Therefore $\boxed{I = (-1, 3]}$