Math 227 Sections 29 and 33 Quiz 8 4/20/18

Name:__________________________

1. Determine whether each series converges or diverges.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n^2}$

$b_n = \frac{\ln n}{n}$ $\quad f(x) = \frac{\ln x}{x^2}$ $\quad f'(x) = \frac{x - 2x\ln x}{x^4}$

$= \frac{1-2\ln x}{x^3}$

$\frac{1-2\ln x}{x^3} < 0$

$1 - 2\ln x < 0$

$\frac{1}{2} < \ln x$

$x > e^{\frac{1}{2}}$

So $b_{n+1} \le b_n$ when $n > e^{\frac{1}{2}}$.

$$\lim_{n\to\infty} b_n = \underbrace{\lim_{n\to\infty} \frac{\ln n}{n^2}}_{\text{type } \frac{\infty}{\infty}} \overset{L'H}{=} \lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{2n}$$

$$= \lim_{n\to\infty} \frac{1}{2n^2} = 0$$

By the AST, $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n^2}$ converges.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\arctan(n)}$

$\displaystyle\lim_{n\to\infty} \frac{1}{\arctan(n)} = \frac{2}{\pi}$, so $\displaystyle\lim_{n\to\infty} \frac{(-1)^{n+1}}{\arctan(n)}$ DNE.

By the TFD, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\arctan(n)}$ diverges.

2. The alternating series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$ converges. Find the smallest n such that you can be sure that the nth partial sum s_n is within $\frac{1}{500}$ of the sum of the series.

$b_n = \frac{1}{n!}$

By the Alternating Series Estimation Theorem,

$|s - s_n| \le b_{n+1} = \frac{1}{(n+1)!}$

$b_5 = \frac{1}{5!} = \frac{1}{120} \not< \frac{1}{500}$

$b_6 = \frac{1}{6!} = \frac{1}{720} < \frac{1}{500}$, so $n+1 = 6$, $\boxed{n = 5}$